Learning, Teaching and Student Engagement

JAMES COOK UNIVERSITY AUSTRALIA

Cairns Singapore Townsville

Mathematics for Chemistry

The module covers concepts such as:

- Unit Conversion
- Shapes
- Algebra
- Percentages/Fractions/Ratios
- Indices
- Scientific Notation
- Significant Figures
- Temperature
- Logarithms

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The Learning Centre

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Mathematics for Chemistry

Contents

1. Foundational Rules of Maths

Common Mathematical Operators

Sign	Meaning	Sign	Meaning
+	Add	=	Equal
-	Subtract Take Away, Subtract, Difference	$\neq$	Not Equal
$\times$	Time, Multiply, Product	$\approx$	Approximately Equal To
$\div$	Divide, Quotient	$>$	Greater Than
$\pm$	Plus and Minus	$\geq$	Greater Than or Equal To
Δ	Delta (Difference Between)	$<$	Less Than
$\sum$	Sigma (Sum of/total)	$\leq$	Less Than or Equal To

Calculations with zeros and ones:

Zeros and ones can often be eliminated. For example:

Any number add or subtract zero is itself, $x + 0 = x$ or $x - 0 = x$

$$(6 + 0 = 6, \qquad 6 - 0 = 6)$$

Any number multiplied by positive 1 stays the same, $x \times 1 = x$ or $\frac{x}{1} = x$

$$(6 \times 1 = 6, \ \ \frac{6}{1} = 6)$$

Note: Using indices (powers), any number raised to the power of zero is 1.

$$\frac{2^2}{2^2} = \frac{4}{4} = 1 \quad or \quad \frac{2^2}{2^2} = 2^{2-2} = 2^0 = 1$$

- Additive Inverse: $x + (-x) = 0$
- Any number multiplied by its reciprocal equals one. $x \times \frac{1}{x} = 1;\ 4 \times \frac{1}{4} = 1$
- Symmetric Property: $If\ x = y\ then\ y = x$
- Transitive Property: $If\ x = y\ and\ y = z, then\ x = z$

  For example, if apples cost \$2 and oranges cost \$2 then apples and oranges are the same price.

Order of Operations

The **Order of Operations** is remembered using the mnemonic known as the BIDMAS or BOMDAS (Brackets, Indices or Other, Multiplication/Division, and Addition/Subtraction).

Brackets	{[( )]}
Indices or **O**ther	$x^2, \sin x, \ln x, etc$
Multiplication or **D**ivision	$\times\ or\ \div$
Addition or **S**ubtraction	+ or -

<u>The Rules:</u>

1. Follow the order (BIMDAS, BOMDAS or BODMAS)
2. If two operations are of the same level, you work from left to right. E.g. $(\times\ or\ \div)\ or\ (+\ or\ -)$
3. If there are multiple brackets, work from the inside set of brackets outwards. {[( )]}

Question 1:

Attempt the following revision examples:

a. $10 - 2 \times 5 + 1 =$

b. $10 \times 5 \div 2 - 3 =$

c. $12 \times 2 - 2 \times 7 =$

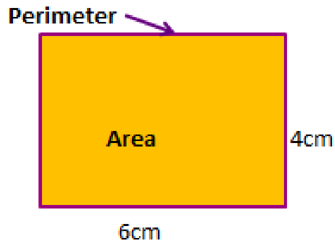
d. $48 \div 6 \times 2 - 4 =$

e. $What\ is\ the\ missing\ operation\ symbol\ 18\ \_\ 3 \times 2 + 2 = 14$

2. Addition & Multiplication Properties

Maths Property	Rule	Example
Commutative Law	$a + b = b + a$ $ab = ba$	$1 + 3 = 3 + 1$ $2 \times 4 = 4 \times 2$
Associative Law	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$	$1 + (2 + 3) = (1 + 2) + 3$ $2 \times (2 \times 3) = (2 \times 2) \times 3$
Distributive Law	$a(b + c) = ab + ac$	$2(3 + 1) = 2 \times 3 + 2 \times 1$
Zero Factor	$a \times 0 = 0$ **If** $ab = 0$**, then either** $a = 0$ **or** $b = 0$	$2 \times 0 = 0$
Rules for Negatives	$-(-a) = a$ $(-a)(-b) = ab$ $-ab = (-a)b = a(-b)$ $= -(ab)$ $(-1)a = -a$	$-(-3) = 3$ $(-2)(-3) = 2 \times 3$ $-2 \times 3 = (-2) \times 3$ $= 2 \times (-3)$ $= -(2 \times 3)$ $(-1) \times 2 = -2$
Rules for Quotients	$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ $\frac{-a}{-b} = \frac{a}{b}$ $If\ \frac{a}{b} = \frac{c}{d}\ then\ ad = bc$	$-\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2}$ $\frac{-6}{-3} = \frac{6}{3}$ $If\ \frac{1}{2} = \frac{3}{4}\ then\ 1 \times 4 = 2 \times 3$

3. Area and Perimeter

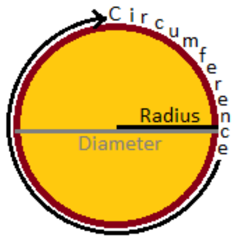


Area measures the amount of space enclosed in a two dimensional shape and is measured in square units. The area of a rectangle or square is calculated by multiplying the length (or breadth) by the height: $\boldsymbol{A} = \boldsymbol{l} \times \boldsymbol{h}$.

The image to the right describes a rectangle with side lengths measured in the unit of distance centimetre (cm). The **area** of the rectangle is calculated by multiplying the base of 6cm by the height of 4cm. $6cm \times 4cm = 24cm^2$

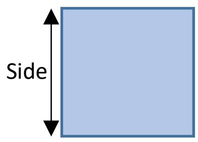
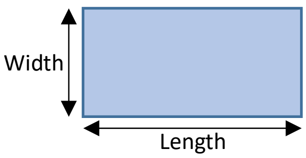
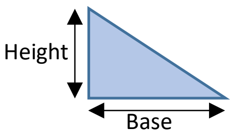
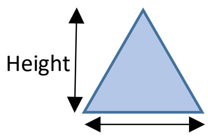
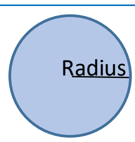
The **perimeter** is the length of the boundary. The can be calculated by adding up the total length for all sides of the shape. To calculate the perimeter of the above rectangle, the lengths of all four sides are added together: $4cm + 4cm + 6cm + 6cm = 20cm$

A **polygon** is any two dimensional shape with straight sides. The formula for the area of any polygon is derived from the area of a rectangle or triangle (a rectangle halved). A small list of area and perimeter formulas are provided below.



A **circle** is a simple enclosed shape with 1 round side where all points on the perimeter are the same distance from the centre. A circle is not a polygon though, and requires a different approach when calculating area and perimeter. A circle is described by its **radius** (the length from centre to one side).The total length from one side to the other, passing through the centre is called the **diameter**. Two radii together add up to one diameter. The perimeter of a circle is known as the **circumference**.

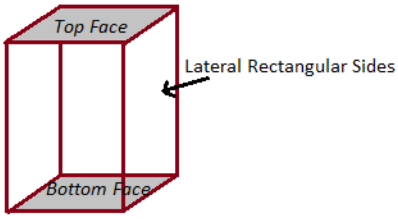
A ratio known as **pi** or the symbol π is used to describe the relationship between the diameter of a circle and its circumference. The symbol π describes a recurring decimal generally shortened to 3.14. The diameter of a circle can wrap around its circumference π (roughly 3 and a bit) times, allowing the circumference of a circle to be determined with the formula **Circumference = $\boldsymbol{\pi}$d or C=2$\boldsymbol{\pi}$r** (since 2 radii = 1 diameter)**.** The formula for the area of a circle also uses π and radius, and can be calculated using $\mathbf{Area} = \boldsymbol{\pi r^2}$.

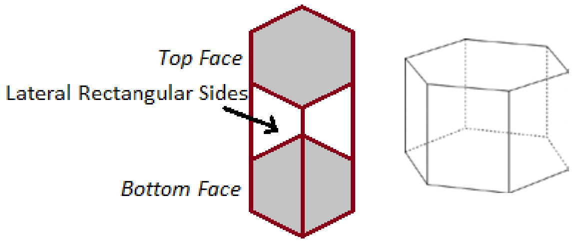
Diagram	Name	Perimeter	Area
	Square (All sides are same length)	$P = 4s$ Perimeter=4 x side	$A = s \times s$ or $A = s^2$ Area = Length of Side x Side
	Rectangle	$P = 2l + 2w$ Perimeter = 2 x Length + 2 x Width	$A = l \times w$ Area = Length x Width
	Right angle Triangle (Sides are different lengths)	$P = s_1 + s_2 + s_3$ Perimeter = the sum of all sides	$A = ½\, b \times h$ Area = ½ Base x Height
	Equalateral Triangle (All sides are equal in length)	$P = 3s$ Perimeter = the sum of all sides	$A = ½\, b \times h$ Area = ½ Base x Height
	Circle	$C = \pi d$ or $C = \pi r$ Circumference = π x Diameter Circumference = 2 x π x Radius	$A = \pi r^2$ Area = π x radius x radius

4. Volume of 3 Dimensional Shapes

Three dimensional (3D) shapes have length, breadth/width and height. The space inside a 3D object is referred to as its volume. Prisms are a group of 3D shapes with a volume that can be calculated using a common rule that will be described further.

Prism: a prism has identical (congruent) parallel faces (top and bottom), that are joined together by lateral rectangular faces. They are a solid consisting of two identical polygons joined at either end by parallel lines (rectangles).



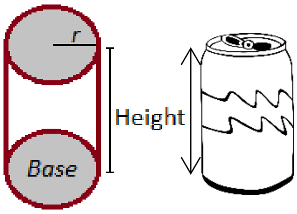


Three dimensional shapes have surface area and volume or capacity:

- Volume/Capacity: the amount of 3D space occupied
  - Liquid volume: Units measured include litres (L) and millilitres (mL)
  - Solid volume: Units measure include cubic centimetres (cm^3) and cubic metres (m^3).

The volume of any prism involves calculating the **area** of the $polygon\ base$ by the $\mathbf{height}$: $\boldsymbol{V = Ah}$. This formula works for any prism. Using a rectangular prism as an example, the length of the base is multiplied by the breadth/width of the base (to calculate the area of the base), then multiplied by the height of the prism (**V=lbh**).

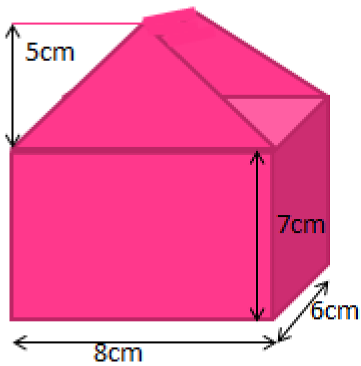
A cylinder doesn't have straight rectangular sides, meaning it is not a prism, but the same formula (**V=Ah**) can be used in calculating its volume. The area of the circular base is calculated using πr^2, then multiplied by the height of the cylinder. Various other 3D shapes also use a variation of the prism volume formula.



Area (of circular base) = πr^2
Volume = Ah

EXAMPLE:

Find the volume of the prism shown:



First, the area of the front face (identical to the rear face) is calculated, then multiplied by the height for $V = Ah$. The front face consists of a triangle and a rectangle. Hence, first two formulas are applied.

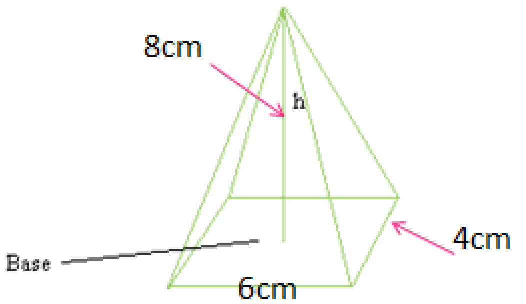
$\boldsymbol{A = \frac{1}{2}(lb) + (lb)}$ (add the areas of a triangle and rectangle)

So, $A = \frac{1}{2}(5 \times 8) + (7 \times 8); A = 20 + 56 = 76 \ \therefore A = 76cm^2$

The area of the base (front face) is then multiplied by the height (depth) $(6cm)$, assuming the prism is on its side: $V = Ah; V = 76 \times 6 = 456 \quad \therefore$ $V = 456cm^3$

The volume of a pyramid and a cone

The volume of a pyramid is a third of the volume of a rectangluar prism with the <u>same height</u> and the <u>same rectangular base</u>. If the base of both shapes were open and filled with water, the prism would take three times the amount of water as the pyramid, hence a ratio of: $volume\ of\ prism :\ volume\ of\ pyramid$ is $3:1$



Adapted from http://www.basic-mathematics.com

To calculate the volume of a pyramid (right) the following formula is used: $\boldsymbol{V = \frac{1}{3}Ah}$ Hence, the area of the pyramid is $24cm^2$

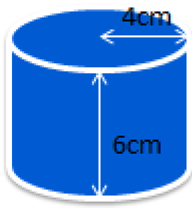
Thus: $V = \frac{1}{3}24 \times 8\ \therefore V = 64cm^3$

Let's check the theory that the volume of a pyramid is one third of the voume of a rectangular prism of the same height and base. $V = lbh; V = 6 \times 4 \times 8 = 192cm^3$; so to check $\frac{192}{3} = 64$✓

The same applies for a cone and a cylinder, the ratio of: $volume\ of\ cylinder :\ volume\ of\ cone$ is $3:1$

Question 3:

Calculate the following:



a. The volume of a cylinder with the radius of $4cm$ and height of $6cm$.

b. The volume of the cone would be if it had a base with a $4cm$ radius and the height of $6cm$.

c. A measuring cylinder has an internal radius of 1cm and a height of 20cm,

i. Calculate the total volume that it can hold in cm^3

ii. The same measuring cylinder is filled to 15cm high with liquid, calculate the volume of liquid in cm^3

5. Fractions – addition, subtraction, multiplication and division

Adding and subtracting fractions draws on the concept of equivalent fractions. The golden rule is that you can only **add and subtract fractions** if they have the **same denominator**, for example,

$$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}.$$

If two fractions do ***not*** have the same denominator, we must use equivalent fractions to find a "common denominator" before they can be added together.

In the example $\frac{1}{4}+\frac{1}{2}$, 4 is the lowest common denominator. Use the equivalent fractions concept to change $\frac{1}{2}\ into\ \frac{2}{4}$ by multiplying both the numerator and denominator by two: $\frac{1}{2}\text{ x }\frac{2}{2}=\frac{2}{4}$

Now that the denominators are the same, addition or subtraction operations can be carried out.

In this example the lowest common denominator is found by multiplying 3 and 5, and then the numerators are multiplied by 5 and 3 respectively:

$$\frac{1}{3}+\frac{2}{5}=\frac{1\times 5}{3\times 5}+\frac{2\times 3}{5\times 3}=\frac{(1\times 5)+(2\times 3)}{(3\times 5)}=\frac{5+6}{15}=\frac{11}{15}$$

Compared to addition and subtraction, *multiplication and division of fractions* is easy to do, but sometimes a challenge to understand how and why the procedure works mathematically. For example, imagine I have $\frac{1}{2}$ of a pie and I want to share it between 2 people. Each person gets a quarter of the pie.

Mathematically, this example would be written as: $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.

Remember that fractions and division are related; in this way, multiplying by a half is the same as dividing by two.

So $\frac{1}{2}$ (two people to share) of $\frac{1}{2}$ (the amount of pie) is $\frac{1}{4}$ (the amount each person will get).

But what if the question was more challenging: $\frac{2}{3}\times\frac{7}{16}=?$ This problem is not as easy as splitting pies.

A mathematical strategy to use is: "Multiply the numerators then multiply the denominators"

Therefore, $\frac{2}{3}\times\frac{7}{16}=\frac{(2\times 7)}{(3\times 16)}=\frac{14}{48}=\frac{7}{24}$

An alternative method you may recall from school is to simplify each term first. Remember, 'What we do to one side, we must do to the other.'

The first thing we do is look to see if there are any common multiples. For $\frac{2}{3}\times\frac{7}{16}=?$ we can see that 2 is a multiple of 16, which means that we can divide top and bottom by 2:

$$\frac{2^{\div 2}}{3}\times\frac{7}{16_{\div 2}}=\frac{1}{3}\times\frac{7}{8}=\frac{1\times 7}{3\times 8}=\frac{7}{24}$$

Division of fractions **seems odd, but it is a simple concept:**

You may recall the expression 'invert and multiply', which means we flip the divisor fraction (second term fraction). Hence, $\div \frac{1}{2}$ *is the same as* $\times \frac{2}{1}$

This 'flipped' fraction is referred to as the **reciprocal of the original fraction**.

Therefore, $\frac{2}{3} \div \frac{1}{2}$ is the same as $\frac{2}{3} \times \frac{2}{1} = \frac{(2\times2)}{(3\times1)} = \frac{4}{3} = 1\frac{1}{3}$ Note: dividing by half doubled the answer.

Question 5:

a) Find the reciprocal of $2\frac{2}{5}$

b) $\frac{2}{3} \times \frac{7}{13} =$

c) $1\frac{1}{6} \times \frac{2}{9} =$

d) $\frac{3}{7} \div \frac{2}{5} =$

e) $2\frac{2}{5} \div 3\frac{8}{9} =$

f) $\frac{(-25)\div(-5)}{4-2\times7} =$

g) $\frac{-7}{2} \div \frac{-4}{9} =$

h) If we multiply 8 and the reciprocal of 2, what do we get?

i) Which is the better score in a physiology test; 17 out of 20 or 22 out of 25?

j) What fraction of H_2O_2 is hydrogen?

k) A chemist prepares a solution containing $\frac{1}{50}$ mole of propanol in 1000ml of water. How many moles are there in a 250ml aliquot of this solution?

More assistance with fraction arithmetic is available at:

https://www.khanacademy.org/math/arithmetic/fraction-arithmetic

6. Converting Decimals & Fractions

Converting Decimals into Fractions

Decimals are an almost universal method of displaying data, particularly given that it is easier to enter decimals, rather than fractions, into computers. But fractions can be more accurate. For example, $\frac{1}{3}$ is not 0.33 it is $0.3\dot{3}$

The method used to convert decimals into fractions is based on the notion of place value. The place value of the last digit in the decimal determines the denominator: tenths, hundredths, thousandths, and so on…

<u>Example problems:</u>

1. 0.5 has 5 in the tenths column. Therefore, 0.5 is $\frac{5}{10} = \frac{1}{2}$ (simplified to an equivalent fraction).
2. 0.375 has the 5 in the thousandth column. Therefore, 0.375 is $\frac{375}{1000} = \frac{3}{8}$
3. 1.25 has 5 in the hundredths column and you have $1\frac{25}{100} = 1\frac{1}{4}$

The hardest part is converting to the lowest equivalent fraction. If you have a scientific calculator, you can use the fraction button. This button looks different on different calculators so read your manual if unsure.

If we take $\frac{375}{1000}$ from example 2 above:

Enter 375 then  followed by 1000 press = and answer shows as $\frac{3}{8}$.

NOTE: The calculator does not work for rounded decimals; especially thirds. For example, $0.333 \approx \frac{1}{3}$

This table lists some commonly encountered fractions expressed in their decimal form:

Decimal	Fraction	Decimal	Fraction
0.125	$\frac{1}{8}$	**0.5**	$\frac{1}{2}$
0.25	$\frac{1}{4}$	**0.66667**	$\frac{2}{3}$
0.33333	$\frac{1}{3}$	**.75**	$\frac{3}{4}$
0.375	$\frac{3}{8}$	**.2**	$\frac{1}{5}$

Question 6:

Convert the following to a fraction (No Calculator first, then check.):

a) 0.65 =   b) 0.54 =   c) 2.666 =   d) 3.14 =

e) What is 40 multiplied by 0.2 (use your knowledge of fractions to solve)

Converting Fractions into Decimals

Converting fractions into decimals is based on place value. Using the concept of equivalent fractions, we can easily convert $\frac{2}{5}$ into a decimal. First we convert to a denominator that has a 10 base:

$\frac{2}{5}$ into tenths $\rightarrow \frac{2^{\times 2}}{5_{\times 2}} = \frac{4}{10}$ $\therefore$ we can say that two fifths is the same as four tenths: 0.4

Converting a fraction to decimal form is a simple procedure because we simply use the divide key on the calculator.
Note: If you have a mixed number, convert it to an improper fraction before dividing it on your calculator.

Example problems:

1. $\frac{2}{3} = 2 \div 3 = 0.66666666666\ldots \approx 0.67$
2. $\frac{3}{8} = 3 \div 8 = 0.375$
3. $\frac{17}{3} = 17 \div 3 = 5.6666666\ldots \approx 5.67$
4. $3\frac{5}{9} = (27 + 5) \div 9 = 3.555555556\ldots \approx 3.56$

Question 7

Convert the following to a decimal (Round your answer to three decimal places where appropriate):

a) $\frac{17}{23} =$

b) $\frac{5}{72} =$

c) $56\frac{2}{3} =$

d) $\frac{29}{5} =$

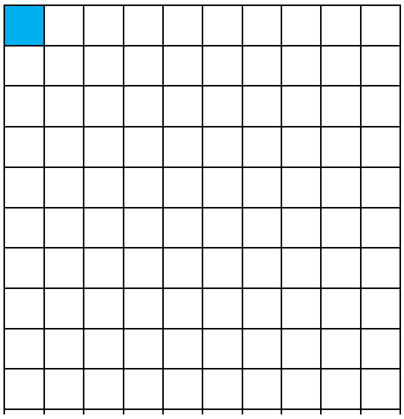
KHANACADEMY

Watch this short Khan Academy video for further explanation:
"Converting fractions to decimals" (and vice versa)
https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/decimal-to-fraction-pre-alg/v/converting-fractions-to-decimals

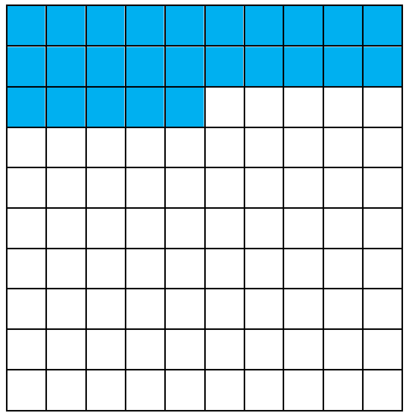
7. Calculating Percentages

A percentage is a number expressed as a fraction of 100. The word *percent* actually means per one hundred.

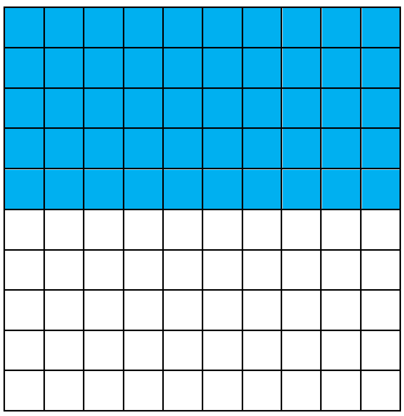
The word *percent* means per 100. It is the number of units in a group of 100 units. Expressed as a fraction $1\% = \frac{1}{100}$, $2\% = \frac{2}{100}$, and $10\% = \frac{10}{100}$.



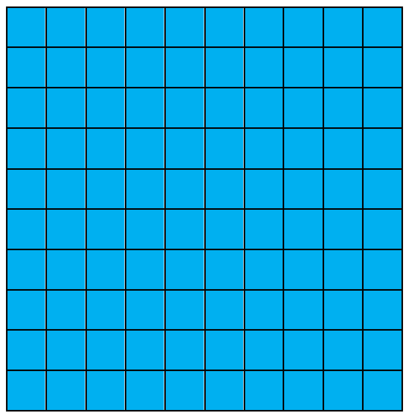
One percent (**1 %**) means 1 per 100 or $\frac{1}{100}$



25 % means 25 per 100 or $\frac{25}{100}$



50 % means 50 per 100 or $\frac{50}{100}$



100 % means 100 per 100 or $\frac{100}{100}$

Items/measurements are rarely found in groups of exactly 100, so we can calculate the number of units that would be in the group if it did contain the exactly 100 units – the percentage.

$$\textit{Percent } (\%) = \frac{number\ of\ units}{total\ number\ of\ items\ in\ the\ group} \times 100$$

Example:

JCU has 2154 female and 1978 male students enrolled. What percentage of the students is female?

The total number of students is 4132, of which 2154 are female.

% female = $\frac{number\ of\ females}{total\ number\ of\ students} \times 100$

$= \frac{2154}{4132} \times 100$

= 52.13 %

When John is exercising his heart rate rises to 180 bpm. His resting heart rate is 70 % of this. What is his resting heart rate?

70 % = $\frac{resting\ heart\ rate}{180} \times 100$

Rearrange:

Start with	$\mathbf{70\ \% = \frac{resting\ heart\ rate}{180} \times 100}$
Divide both sides by 100	$\mathbf{\frac{70}{100} = \frac{resting\ heart\ rate}{180}}$
Multiply both sides by 180	$\mathbf{\frac{70}{100} \times 180 = resting\ heart\ rate}$

Resting heart rate = $\frac{70}{100} \times 180$

= 126 bpm

Question 4:

Calculate the following:

(i) Sally bought a television that was advertised for $467.80. She received a discount of $32.75. What percentage discount did she receive?

(ii) 50 kg of olives yields 23 kg of olive oil. What percentage of the olives' mass was lost during the extraction process?

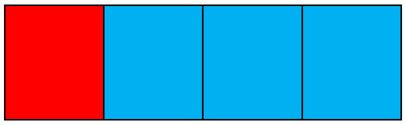
(iii) The recommended daily energy intake for women is 8700 KJ. What percentage over the recommended intake is a women consuming if her energy intake is 9500 KJ?

(iv) An advertisement the fruit and vegetable shop states that there is 25 % off everything. Bananas are normally $22/Kg, what is the new per kilo price?

8. Ratios

A ratio is a comparison of the size of one number to the size of another number. A ratio represents for every determined amount of one thing, how much there is of another thing. Ratios are useful because they are unit-less. That is, the relationship between two numbers remains the same regardless of the units in which they are measured.

Ratios use the symbol **:** to separate quantities being compared. For example, 1:3 means 1 unit to 3 units.



There is 1 red square to 3 blue squares

1:3

1 to 3

Ratios can be expressed as fractions but you can see from the above diagram that 1:3 is not the same as $\frac{1}{3}$.
The fraction equivalent is $\frac{1}{4}$

Example:

A pancake recipe requires flour and milk to be mixed to a ratio of 1:3. This means one part flour to 3 parts milk. No matter what device is used to measure, the ratio must stay the same.

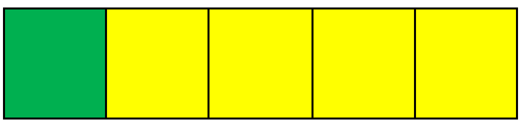
So if I add 200 mL of flour, I add 200 mL x 3 = 600 mL of milk

If I add 1 cup of flour, I add 3 cups of milk

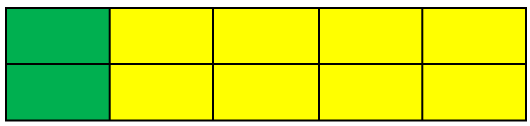
If I add 50 grams of flour, I add 150 grams of milk

Scaling ratios

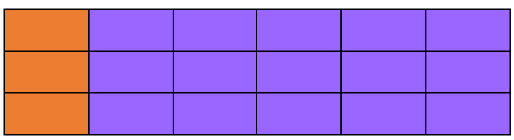
A ratio can be scaled up:



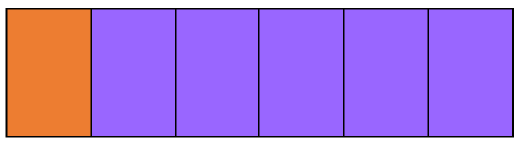
1:4 = 2:8



Or scaled down:



3:15 = 1:5



1:5 is the same as 2:10 is the same as 3:15 is the same as 4:20 and so on

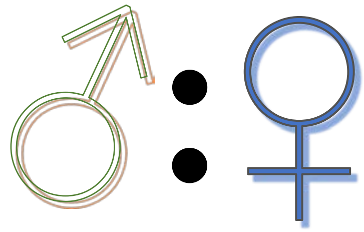
Scaling ratios is useful in the same way that simplifying fractions can be helpful, for example, in comparing values. For ratios the same process as simplifying fractions is applied – that is, scaling must be applied to both numbers.

For example, a first year chemistry subject has 36 males and 48 females, whereas the mathematics subject has 64 males and 80 females. You are asked to work out which cohort has the largest male to female ratio.

The male: female ratios can be expressed as:

36:48 – chemistry subject

64:80 – mathematics subject



Both numbers of the ratio 36:48 can be divided by 12 to leave the ratio 3:4
Both numbers of the ratio 64:80 can be divided by 16 to leave the ratio 4:5

These two ratios cannot be easily directly compared, but they can be rescaled to a common value of 20 for the females, against which the males can be compared if they are rescaled equivalently.

3 (x5):4 (x5) = 15:20 – chemistry subject

4 (x4):5 (x4) = 16:20 – mathematics subject

Comparing the ratios now shows that the chemistry subject has a slightly higher ratio of males to females.

Question 8:

Calculate the following:

(i) Jane reads 25 pages in 30 minutes. How long does it take her to read 200 pages?

(ii) John uses 7 eggs to make 4 muffins. How many eggs does he need to make 12 muffins?

(iii) Jane is swimming laps at the local swimming pool. She swims 4 laps in 3 minutes. How long does it take her to swim 20 laps?

Molar Ratios:

Ratios are used in **stoichiometry** – the study of mass relationships in chemical reactions. Consider the following equation:

$$2HCl_{(aq)} + CaO_{(s)} \rightarrow CaCl_{2(aq)} + H_2O_{(l)}$$

This equation is showing that 2 HCl molecules react with 1 CaO molecule to form 1 $CaCl_2$ molecule and 1 H_2O molecule.

$$2HCl_{(aq)} + CaO_{(s)} \rightarrow CaCl_{2(aq)} + H_2O_{(l)}$$

$$2 \quad : \quad 1 \quad : \quad 1 \quad : \quad 1$$

Example:

How many molecules of CaO would you need to react with 24 molecules of HCl?

$$2HCl_{(aq)} + CaO_{(s)} \rightarrow CaCl_{2(aq)} + H_2O_{(l)}$$

$$2 \quad : \quad 1 \quad : \quad 1 \quad : \quad 1$$

The reaction between HCl and CaO uses 2 HCl molecules for every 1 CaO molecule.

Ratio is 2 HCl : 1 CaO

24 HCl : 12 CaO

12 molecules of CaO is required.

Question 9:

Calculate the following:

(i) $H_2SO_{4(aq)} + 2LiOH_{(aq)} \rightarrow LiSO_{4(aq)} + 2H_2O_{(l)}$
How many molecules of H_2SO_4 are required to react with 8 molecules of LiOH?

(ii) $3MnO_{2(s)} + 4Al_{(s)} \rightarrow 2Al_2O_{3(s)} + 3Mn_{(s)}$
How many molecules of Al_2O_3 are produced if we have 12 molecules of MnO_2?

(iii) $2KClO_{3(s)} \rightarrow 2KCl_{(s)} + 3O_{2(g)}$
How many molecules of $KClO_3$ are required to produce 27 molecules of O_2?

**Refer to Maths Module 3: Ratio, Proportion and Percent (p.3-4) for further assistance with ratios*

9. Unit Conversion

Measurements consist of two parts – the number and the identifying unit.

Number → **50 kg** ← Identifying unit

In scientific measurements, units derived from the metric system are the preferred units. The metric system is a decimal system in which larger and smaller units are related by factors of 10.

Table 1: Common Prefixes of the Metric System

Prefix	Abbreviation	Relationship to Unit	Exponential Relationship to Unit	Example
mega-	M	1 000 000 x Unit	10^{6} x Unit	2.4ML -Olympic sized swimming pool
kilo-	k	1000 x Unit	10^{3} x Unit	The average newborn baby weighs 3.5kg
-	-	Units	Unit	meter, gram, litre, sec
deci-	d	1/10 x Unit **or** 0.1 x Unit	10^{-1} x Unit	2dm - roughly the length of a pencil
centi-	c	1/100 x Unit **or** 0.01 x Unit	10^{-2} x Unit	A fingernail is about 1cm wide
milli-	m	1/1000 x Unit **or** 0.001 x Unit	10^{-3} x Unit	A paperclip is about 1mm thick
micro-	µ	1/1 000 000 x Unit **or** 0.000001 x Unit	10^{-6} x Unit	human hair can be up to 181 µm
nano-	n	1/1 000 000 000 x Unit **or** 0.000000001 x Unit	10^{-9} x Unit	DNA is 5nm wide

Table 2: Common Metric Conversions

Unit	Larger Unit	Smaller Unit
1 metre	1 kilometre = 1000 meters	100 centimetres = 1 meter 1000 millimetres = 1 meter
1 gram	1 kilogram = 1000 grams	1000 milligrams = 1 gram 1 000 000 micrograms = 1 gram
1 litre	1 kilolitre = 1000 litres	1000 millilitres = 1 litre

Example:

Convert 0.15 g to kilograms and milligrams	**Convert 5234 mL to litres**
Because 1 kg = 1000 g, 0.15 g can be converted to kilograms as shown: $\mathbf{0.15\ g\ x\ \frac{1\ kg}{1000\ g} = 0.00015\ kg}$ Also, because 1 g = 1000 mg, 0.15 g can be converted to milligrams as shown: $\mathbf{0.15\ \cancel{g}\ x\ \frac{1000\ mg}{1\ \cancel{g}} = 150\ mg}$	Because 1 L = 1000 mL, 5234 mL can be converted to litres as shown: $\mathbf{5234 mL\ x\ \frac{1\ L}{1000\ mL} = 5.234\ L}$

Question 2:

Convert the following measurements:

1. **Convert 600 g to kilograms and milligrams**

2. **Convert 4.264 L to kilolitres and millilitres**

3. **Convert 670 cm to meters and kilometres**

10. Introduction to Powers

Powers are a method of simplifying expressions.

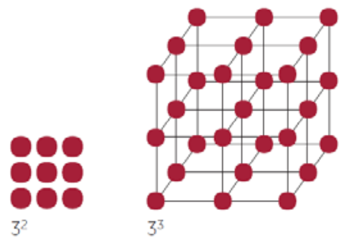
- An equation such as : $7+7+7+7+7+7+7+7+7=63$ could be simplified as: $7 \times 9 = 63$
- Whereas an expression such as: $7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$ could be simplified as: 7^9

A simple way to describe powers is to think of them as how many times the base number is multiplied by itself.

- $5 \times 5 \times 5 \times 5$ is the same as 5^4 is the same as 625

$$5^4$$

- The 5 is called the base.
- The 4 is called the exponent, the index or the power.



- The most common way to describe this expression is, 'five to the power of four.'
- The two most common powers (2 & 3) are given names: 3^2 is referred to as '3 squared' and 3^3 as '3 cubed.'
- However, note that -3^2 is different to $(-3)^2$; the first is equivalent to (-9), whereas the second is equivalent to $(+9)$ In the first example, -3^2, only the 3 is raised to the power of two, in the second, $(-3)$is rasied to the power of two, so $(-3) \times (-3) = 9$, 'positive' number because $(-) \times (-) = (+)$
- Whereas, a negative power, such as 6^{-3}, reads as 'six raised to the power of negative three', it is the same as $\frac{1}{6^3}$: it is the reciprocal. $\therefore\ 6^{-3} = \frac{1}{6^3} = \frac{1}{216}$
- To raise to the negative power means one divided by the base raised to the power, For example, 6^{-3} can be read as 'one divided by the 6 raised to the power of three' (six cubed).
- Another example: $\frac{1}{4^3}$ is the same as 4^{-3} , which is also $\frac{1}{64}$
- Take care when working with powers of negative expressions: $(-2)^3 = (-8)\ negative\ times\ negative\ times\ negative = negative\ (- \times - \times - = -)$
  $(-2)^4 = 16\ (- \times -)(- \times -) = +$
  $\therefore\ (-x)^5$ will be a negative expression, whereas $(-x)^6$ will be positive.

It helps if you can recognise some powers, especially later when working with logarithms

* The powers of 2 are: 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 2048, 4096, 8192,…
* The powers of 3 are: 3, 9, 27, 81, 243, 729,…
* The powers of 4 are every second power of 2
* The powers of 5 are: 5, 25, 125, 625, 3125,…
* The powers of 6 are: 6, 36, 216, 1296,…
* The powers of 7 are: 7, 49, 343, 2401,…
* The powers of 8 are: every third power of 2
* The powers of 9 are every second power of 3
* The powers of 10 are: 10, 100, 1000, 10 000, 100 000, 1 000 000,…
* The powers of 16 are every fourth power of 2

Question 10:

Work out the value of the following based on what you have understood from the introduction:

a. $2^3 =$

b. $9^2 =$

c. $\left(\frac{1}{2}\right)^3 =$

d. $(-4)^2 =$

e. $(-3)^3$

f. $(-\frac{1}{4})^2 =$

g. $0^3 =$

h. $(-0.1)^2 =$

i. $(0.5)^3 =$

j. $4^{-2} =$

k. $1^{-11} =$

l. $4^{-1} =$

m. $(-4)^{-1} =$

n. $(0.5)^{-4}=$

o. $(\frac{3}{4})^{-3} =$

p. $(-2)^{-3} =$

11. Power Operations

Powers are also called **indices**; we can work with the **indices** to simplify expressions and to solve problems.

Some key ideas:

- Any base number raised to the power of 1 is the base itself: for example, $5^1 = 5$
- Any base number raised to the power of 0 equals 1, so: $4^0 = 1$
- Powers can be simplified if they are **multiplied** or **divided** and have the **same** base.
- Powers of powers are multiplied. Hence, $(2^3)^2 = 2^3 \times 2^3 = 2^6$
- A negative power indicates a reciprocal: $3^{-2} = \frac{1}{3^2}$

$$\begin{gathered}
\boldsymbol{a^0 = 1} \\
\boldsymbol{a^1 = a} \\
\boldsymbol{a^n \times a^m = a^{m+n}} \\
\boldsymbol{\frac{a^n}{a^m} = a^{n-m}} \\
\boldsymbol{(a^n)^m = a^{nm}} \\
\boldsymbol{(ab)^n = a^n b^n} \\
\boldsymbol{a^{-n} = \frac{1}{a^n}} \\
\boldsymbol{a^{\frac{1}{n}} = \sqrt[n]{a}} \\
\boldsymbol{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m}
\end{gathered}$$

Certain rules apply and are often referred to as: **Index Laws.**

The first rule: $a^m \times a^n = a^{m+n}$

- To multiply powers of the same base, add the indices
- How does this work?
- Let's write out the 'terms'
- $a^7 \times a^2 = a^{7+2} = a^9$

$$\overbrace{(a \times a \times a \times a \times a \times a \times a)}^{7} + \overbrace{(a \times a)}^{2}$$

The second rule: $\frac{x^9}{x^6} = x^{9-6} = x^3$

- To divide powers of the same base, subtract the indices
- How does this work?
- We are dividing so we can cancel
- Subtract 6 from 9 (numerator) and six from six (denominator)
- $\therefore \frac{x^9}{x^6} = x^{9-6} = x^3$
- From the second law we learn why x^0=1
- Any expression divided by itself equals 1
- so $\frac{x^3}{x^3} = 1$ or x^{3-3}=x^0 which is 1

$$\frac{x \times x \times x \times x \times x \times x \times x \times x \times x}{x \times x \times x \times x \times x \times x} = \text{apply cancel method}$$

$$\frac{x \times x \times x \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{\cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}} = \frac{x \times x \times x}{1}$$

$$= x \times x \times x = x^3$$

The third rule: $(b^a)^m$=b^{am}

- To raise a power to a power, multiply the indices.
- $(b^2)^3$
- How does this work?
- $(b \times b) \times (b \times b) \times (b \times b) = b^6$
- Therefore, we multiply the indices.

The fourth rule: $(xy)^n$ = $x^n y^n$

- A power of a product is the product of the powers.
- $(3 \times 4)^2$
- How does this work?
- $(3 \times 4) \times (3 \times 4) = 3^2 \times 4^2$
- $12 \times 12 = 9 \times 16 = (both\ equal\ 144)$
- Therefore, we can expand (remove the brackets).

The fifth rule: $(^a/_b)^m = \frac{a^m}{b^m}$ (as long as *b* is not zero)

- A power of a quotient is the quotient of the powers.
- $(^5/_2)^2 = \frac{5^2}{2^2}$How does this work?
- $(5 \div 2) \times (5 \div 2) = 5^2 \div 2^2$
- $2.5 \times 2.5 = \frac{25}{4} = (both\ equal\ 6\frac{1}{4})$

Therefore, we can expand (remove the brackets)

Fractional Indices: $a^{\frac{m}{n}} = (a^{\frac{1}{n}})^m$

- This is related to the third index law and will be explained later in 'Fractional Powers'.

Below is a summary of the index rules:

Index rule	Substitute variables for values
$a^m \times a^n = a^{m+n}$	$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
$a^m \div a^n = a^{m-n}$	$3^6 \div 3^3 = 3^{6-3} = 3^3 = 27$
$(a^m)^n = a^{mn}$	$(4^2)^5 = 4^{2\times5} = 4^{10} = 1048\,576$
$(ab)^m = a^m b^m$	$(2\times5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100$
$(^a/_b)^m = a^m \div b^m$	$(10 \div 5)^3 = 2^3 = 8;\ (10^3 \div 5^3) = 1000 \div 125 = 8$
$a^{-m} = \frac{1}{a^m}$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$a^{\frac{1}{m}} = \sqrt[m]{a}$	$8^{1/3} = \sqrt[3]{8} = 2$
$a^0 = 1$	$6^3 \div 6^3 = 6^{3-3} = 6^0 = 1; (6 \div 6 = 1)$

EXAMPLE PROBLEMS:

1. Simplify $6^5 \times 6^3 \div 6^2 \times 7^2 + 6^4 =$
   $= 6^{5+3-2} \times 7^2 + 6^4$
   $= 6^6 \times 7^2 + 6^4$
2. Simplify $g^5 \times h^4 \times g^{-1} =$
   $= g^5 \times g^{-1} \times h^4$
   $= g^4 \times h^4$

Question 11:

Apply the index laws/rules:

a. Simplify $5^2 \times 5^4 + 5^2 =$

b. Simplify $x^2 \times x^5 =$

c. Simplify $4^2 \times \mathrm{t}^3 \div 4^2 =$

d. Simplify $(5^4)^3 =$

e. Simplify $\frac{2^4 3^6}{3^4} =$

f. Simplify $3^2 \times 3^{-5} =$

g. Simplify $\frac{9(x^2)^3}{3xy^2} =$

h. Simplify $a^{-1}\sqrt{a} =$ (applies to section 7)

What is the value of x for the following?

i. $49 = 7^x$

j. $\frac{1}{4} = 2^x$

k. $88 = 11^1 \times 2^x$

l. $480 = 2^x \times 3^1 \times 5^1$

m. Show that $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$

Use index laws to simplify the following, and write using scientific notation

a. $(3 \times 10^2) \times (2 \times 10^4)$

b. $(8 \times 10^6) \div (4 \times 10^2)$

c. $(7 \times 10^2) \times (8 \times 10^2)$

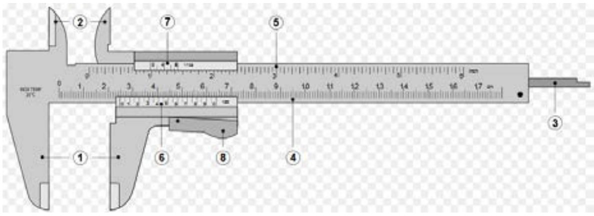
d. $(6 \times 10^4) \times (0.5 \times 10^2)$

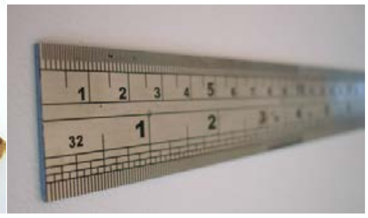
e. $(3 \times 10^{-4}) \div (3 \times 10^{-5})$

f. $(8.8 \times 10^{-1}) \times (8.8 \times 10^{-1})$

13. Significant Figures

Scientific notation is used for scientific measurements as is **significant figures**. Both scientific notation and significant figures are used to indicate the accuracy of measurement. For example, if we measured the length of an item with an old wooden ruler, a steel ruler, and then some steel callipers, the measurements would vary slightly in the degree of accuracy.





Perhaps the item might be approximately 8cm long. Hence:

- The wooden ruler will measure to the nearest cm $\pm 0.5cm$, 8.2cm; 2 Significant Figures (sig. figs.)
- The steel ruler will measure to the nearest 0.1 cm $\pm 0.05cm$, perhaps 8.18cm; 3 sig. figs.
- The callipers may measure to the nearest 0.01cm $\pm 0.005cm$, for instance 8.185cm; 4 sig.figs.

Some rules apply:

1. Any non-zero digit is significant. The position of the decimal point does not matter. For example:

14.8	3 sig. figs
148	3 sig. figs
1.48	3 sig. figs
0.148	3 sig. figs
1.4869	5 sig. figs

2. If there are zeros between numbers, they are significant. For example:

1.08	3 sig. figs
10.48	4 sig. figs
1.0048	5 sig. figs
$505.01 = 5.0501 \times 10^2$	5 sig. figs

3. Zeros that are at the right hand end of numbers are not significant, unless stated. For example:

80	1 sig. fig
800	1 sig. fig
8900	2 sig. figs
80090	4 sig. figs
$990 = 9.9 \times 10^2$	2 sig. figs

4. Zeros to the left hand end of decimal numbers are not significant. For example:

0.8	1 sig. fig
0.148	3 sig. figs
0.0048	2 sig. figs
0.5048	4 sig. figs
$0.0006 = 6 \times 10^{-4}$	1 sig. fig

5. Zeros at the right hand end of the decimal are significant. For example:

14.0	2 sig. figs
1.40	3 sig. figs
0.1480	4 sig. figs
1.4800	5 sig. figs
140.0 = 1.4×10^2	4 sig. figs

Consider the measurement 6.54cm, which is a value with 3 significant figures. This implies that the first two digits may be accurate and the last digit is a close estimate. So the measurement of 6.54cm means that the 65mm are accurate but the 0.04mm is an estimate give or take 0.005mm.

Question 13:

These practise examples will help to draw together your understanding about Scientific Notation and significant figures.

1. Write each of the following in scientific notation
2. State the number for significant figures in the value (you might like to refer to the rules).

a. 354

b. 18932

c. 0.0044

d. 0.000506

e. 960

f. 30000

g. 150.900

12. Scientific Notation

Numbers as multiples or fractions of ten	Number	Number as a power of ten
10 x 10 x 10	1000	10^3
10 x 10	100	10^2
10	10	10^1
10 x 1/10	1	10^0
1/10	0.1	10^{-1}
1/100	0.01	10^{-2}
1/1000	0.001	10^{-3}

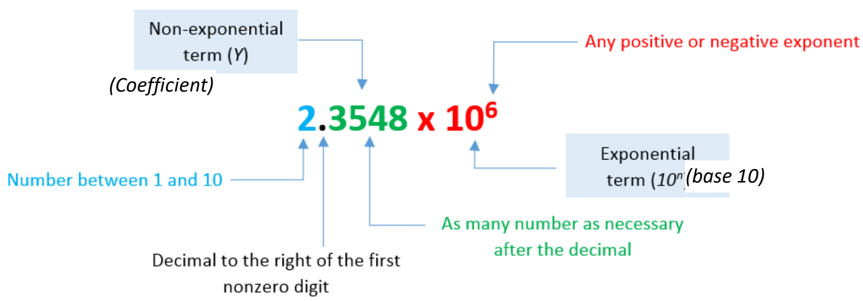
Scientific notation is a convenient method of representing and working with very large and very small numbers. Transcribing a number such as 0.000000000000082 or 5480000000000 can be frustrating since there will be a constant need to count the number of zeroes each time the number is used. Scientific notation provides a way of writing such numbers easily and accurately.

Scientific notation requires that a number is presented as a non-zero digit followed by a decimal point and then a power (exponential) of base 10. The exponential is determined by counting the number places the decimal point is moved.

The number 65400000000 in scientific notation becomes 6.54×10^{10}.

The number 0.00000086 in scientific notation becomes 8.6×10^{-7}.

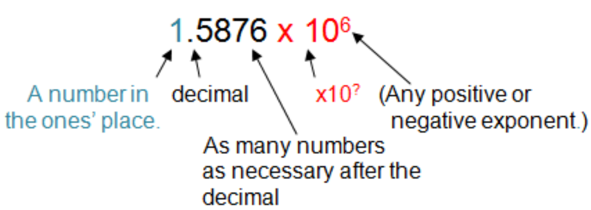
(Note: $10^{-6} = \frac{1}{10^6}$.)

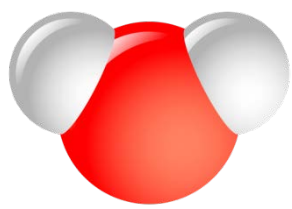


If *n* is positive, shift the decimal point that many places to the right.

If *n* is negative, shift the decimal point that many places to the left.

A classic example of the use of scientific notation is evident in the field of chemistry. The number of molecules in 18 grams of water is 602 000 000 000 000 000 000 000 which is written as 6.02×10^{23} which is much easier to read and verbalise.





Question 12:

Write the following in scientific notation:

a. 450

b. 90000000

c. 3.5

d. 0.0975

Write the following numbers out in full:

e. 3.75×10^2

f. 3.97×10^1

g. 1.875×10^{-1}

h. -8.75×10^{-3}

Calculations with Scientific Notation

Multiplication and *division* calculations of quantities expressed in scientific notation follow the index laws since they all they all have the common base, i.e. base 10.

Here are the steps:

Multiplication	Division
1. **Multiply** the coefficients	1. **Divide** the coefficients
2. **Add** their exponents	2. **Subtract** their exponents
3. Convert the answer to scientific Notation	3. Convert the answer to scientific Notation
Example: $(\mathbf{7.1} \times \mathbf{10^{-4}}) \times (\mathbf{8.5} \times \mathbf{10^{-5}})$ $\mathbf{7.1} \times \mathbf{8.5} = \mathbf{60.35}$ (multiply coefficients) $\mathbf{10^{-4}} \times \mathbf{10^{-5}} = \mathbf{10^{(-4+-5)=-9}}$ (add exponents) $= \mathbf{60.35} \times \mathbf{10^{-9}}$ – check it's in scientific notation ✗ $= \mathbf{6.035} \times \mathbf{10^{-8}}$ – convert to scientific notation ✓	**Example:** $(9 \times 10^{20}) \div (3 \times 10^{11})$ $9 \div 3 = 3$ (divide coefficients) $10^{20} \div 10^{11} = 10^{(20-11)=9}$ (subtract exponents) $= 3 \times 10^9$ – check it's in scientific notation ✓

Recall that addition and subtraction of numbers with exponents (or indices) requires that the base and the exponent are the same. Since all numbers in scientific notation have the same base 10, for *addition* and *subtraction* calculations, we have to adjust the terms so the exponents are the same for both. This will ensure that the digits in the coefficients have the correct place value so they can be simply added or subtracted.

Here are the steps:

Addition	Subtraction
1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent	1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent
2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places	2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places
3. Add the new coefficients	3. Subtract the new coefficients
4. Convert the answer to scientific notation	4. Convert the answer to scientific notation
Example: $(3 \times 10^2) + (2 \times 10^4)$ $4 - 2 = 2$ increase the small exponent by 2 to equal the larger exponent 4 0.03×10^4 the coefficient of the first term is adjusted so its exponent matches that of the second term $= (0.03 \times 10^4) + (2 \times 10^4)$ the two terms now have the same base and exponent and the coefficients can be added $= 2.03 \times 10^4$ check it's in scientific notation ✓	**Example:** $(5.3 \times 10^{12}) - (4.224 \times 10^{15})$ $15 - 12 = 3$ increase the small exponent by 3 to equal the larger exponent 15 0.0053×10^{15} the coefficient of the firs t term is adjusted so its exponent matches that of the second term $= (0.0053 \times 10^{15}) - (4.224 \times 10^{15})$ the two terms now have the same base and exponent and the coefficients can be subtracted. $= -4.2187 \times 10^{15}$ check it's in scientific notation ✓

14. Logarithms

4^2

i. The 4 is called the **base**.

ii. The 2 is called the **exponent, index** or **power**.

$$a^n \times a^m = a^{n+m}$$

$$\frac{a^n}{a^m} = a^{n-m}$$

$$(a^n)^m = a^{nm}$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

$$a^0 = 1$$

This is a summary of some index laws for your reference

$$(ab)^n = a^n b^n$$

$X^0 = 1$

$X^1 = 0$

Roots are used to find an unknown base in a power calculation. For instance, $x^3 = 64$ is the same as $\sqrt[3]{64} = x$; (x is the base).

A **logarithm** is used to find an unknown **power/exponent**. For example, $4^x = 64$ is the same as $\log_4 64 = x$

This example above is spoken as: 'The logarithm of 64 with base 4 is x.' The base is written in subscript.

The general rule is: $\boldsymbol{y = b^x \Leftrightarrow \log_b y = x}$

In simple terms a logarithm attempts to answers how many of one number do we multiply to get another number?

For example how many 3's do we multiply to get 81? The answer is 3x3x3x3 in other words we need 4 of the 3's to get 81. This means that the logarithm is 4

$$\log_3(81) = 4$$

- In mathematics the base can be any number, but only two types are commonly used:
  - $\log_{10} N$ $(base\ 10)$ is often abbreviated as simply Log, and
  - $\log_e N$ $(base\ e)$ is often abbreviated as Ln or natural log
- $\log_{10} N$ is easy to understand for: $\log_{10} 1000 = \log 1000 = 3$ $(10^3 = 1000)$
  $$\log 100 = 2$$
- Numbers which are not 10, 100, 1000 and so on are not so easy. For instance, $\log 500 = 2.7$ It is more efficient to use the calculator for these types of expressions.

$$log_b(x \times y) = log_b(x) + log_b(y)$$
$$log_b\left(\frac{x}{y}\right) = log_b(x) - log_b(y)$$
$$log_b(x^y) = y \times log_b(x)$$

Let's explore further

Logs to the base 10 are used in chemistry where very small concentrations are involved. For instance, acidity is determined by the concentration of H^+ ions (measured in moles/litre and written as $[H^+]$ in a solution. In pure water $[H^+]$ = 0.0000001 mole/litre of 1×10^{-7}

$[H^+]$ = 10^{-14} 10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1}

less acidic More acidic

neutral

A special scale called pH has been developed to measure acidity and it is simply the 'negative index' of the above scale. You may have noticed this scale if you help out with the balancing a swimming pool to ensure the water is safe for swimming.

pH = 14 13 12 11 10 9 8 7 6 5 4 3 2 1

less acidic More acidic

neutral

So a solution with a pH=7 is neutral, while pH =3 is acidic, pH=2 is more acidic, pH 12 is alkaline and so on.

Question 14

Use the scales above to answer these questions.

Convert these $[H^+]$ concentrations to scientific notation and then pH.

a. $[H^+]$ = 0.001 moles/litre

b. $[H^+]$ = 0.00000001 moles/litre

c. $[H^+]$ = 0.0000001 moles/litre

d. $[H^+]$ = 0.000000000001 moles/litre

e. When converting from the $[H^+]$ scale to the pH scale, we take "the negative of the log to base 10 of the $[H^+]$." Can you translate this English statement into a mathematical equation?

Question 15:

Simplify the following

a. $\log 3 + \log 4$

b. $\log_4 3 + \log_4 3 + \log_4 3$

c. pH = $-\log_{10}[H^+]$ if $[H^+] = 0.1$ what is the pH?

d. If the solution in part c above is diluted by a factor of 10 what will be the pH now?

e. If the pH is 3.30 what is $[H^+]$?

15. Solving Equations

The equal sign of an **equation** indicates that both sides of the equation are equal. The equation may contain an **unknown quantity** (or variable) whose value can be calculated. In the equation, $5x + 10 = 20$, the unknown quantity is x. This means that 5 multiplied by something (x) and added to 10, will equal 20.

- To solve an equation means to find all values of the unknown quantity so that they can be substituted to make the left side and right sides **equal**
- Each such value is called a **solution** (e.g. $x = 2$ in the equation above)
- The equation is rearranged and solved in reverse order of operation (SADMOB) to find a value for the unknown

  Eg. $5x + 10 = 20$ **First rearrange by subtracting 10 from the LHS and the RHS**

  $5x + 10\,(-10) = 20(-10)$

  $5x = 10$

  $\frac{5x}{5} = \frac{10}{5}$ **Divide both LHS and RHS by 5**

  $x = 2$

  To check the solution, substitute x for 2 ; $(5 \times 2) + 10 = 20 \therefore 10 + 10 = 20$

Four principles to apply when solving an equation:

1. **Work towards solving the variable**: $(x =)$
2. **Use the opposite mathematical operation**: Remove a constant or coefficient by doing the opposite operation on both sides:

   Opposite of $\times$ is $\div$ Opposite of x^2 is $\sqrt{x}$

   Opposite of $+$ is – Opposite of $\sqrt{x}$ is $\pm x^2$
3. **Maintain balance**: "What we do to one side, we must do to the other side of the equation."
4. **Check:** Substitute the value back into the equation to see if the solution is correct.

ONE-STEP EQUATIONS

Addition	Subtraction
$x + (-5) = 8$ $x + (-5) - (-5) = 8 - (-5)$ $x = 8 + 5$ $\therefore x = 13$	$x - 6 + 6 = (-4) + 6$ So $x = (-4) + 6$ $\therefore x = 2$
Check by substituting 13 for x $13 + (-5) = 8$ $13 - 5 = 8$ $8 = 8$ ✓	Check by substituting 2 for x $2 - 6 = (-4)$ ✓$-4 = -4$

Question 16:

Solve for x:

a. $x + 6 - 3 = 18$

b. $7 = x + (-9)$

c. $x - 12 = (-3)$

d. $18 - x = 10 + (-6)$

Two-step Equations

The following equations require two steps to single out the variable.

Addition Example: $2x + 6 = 14$

Step 1: The constant 6 is subtracted from both sides, creating the following equation:

$$2x + 6 - 6 = 14 - 6 \qquad \textbf{(The opposite of } +6 \textbf{ is } -6\textbf{)}$$
$$2x = 8$$

Step 2: Next both sides are divided by two, creating the following equation:

$$\frac{2x}{2} = \frac{8}{2} \qquad \textbf{(The opposite of } 2x \textbf{ is } \div 2\textbf{)}$$
$$\therefore x = 4$$

Check. Substitute x = 4 into the equation: $2 \times 4 + 6 = 14$

$$8 + 6 = 14$$
$$14 = 14 \quad ✓ \qquad \textbf{(The answer must be correct)}$$

Subtraction Example:

Solve for *j*: $3j - 5 = 16$

Step 1: $3j - 5 = 16$ (deal with the subtraction first)

$3j - 5 + 5 = 16 + 5$ (Opposite of -5 is $+5$)

Thus, $3j = 21$ (the multiply second)

Step 2: $\frac{3j}{3} = \frac{21}{3}$ (the opposite of $\times 3$ is $\div 3$)

$\therefore\ j = 7$

Check: $3 \times 7 - 5 = 16$ ✓

Multi-step Example:

Solve for *T*: $\mathbf{\frac{3T}{12} - 7 = 6}$

$$\mathbf{\frac{3T}{12} - 7 = 6}$$
$$\frac{3T}{12} - 7 + 7 = 6 + 7$$
$$\frac{3T}{12} = 13$$
$$\frac{3T}{12} \times 12 = 13 \times 12$$
$$3T = 156$$
$$\frac{3T}{3} = \frac{156}{3}$$
$$\therefore T = 52$$

Check: $(3 \times 52) \div 12 - 7 = 6$ ✓

Question 17:

Solve the following to calculate the unknown variable:

a. $5x + 9 = 44$

b. $\frac{x}{9} + 12 = 30$

c. $3y + 13 = 49$

d. $4x - 10 = 42$

e. $\frac{x}{11} + 16 = 30$

16. Rearranging Formulas

A formula uses symbols and rules to describe a relationship between quantities. In mathematics, formulas follow the standard rules for algebra and can be rearranged as such. If values are given for all other variables described in the formula, rearranging allows for the calculation of the unknown variable. This is a mathematical application of working with variables and unknowns in physics and engineering problems. In this section, rearranging equations and substituting values is practiced with common physics formulas to determine unknowns. The next section applies these skills to problem solving including application of appropriate units.

Basic Rearranging Example:

Calculate the density $(\boldsymbol{\rho})$ of Lithium

Given: Mass (m) = 268 Volume (v) = 0.5

$$\boldsymbol{\rho} = \frac{m}{V}$$

$$\boldsymbol{\rho} = \frac{268}{0.5}$$

$$\boldsymbol{\rho} = 536$$

Alternatively, calculate the Mass (m) of Lithium

Given: Density (ρ) = 536 and Volume (V) = 0.5

$$\rho = \frac{\boldsymbol{m}}{V}$$

$V \times \rho = \frac{\boldsymbol{m} \times \cancel{V}}{\cancel{V}}$ (rearrange to calculate m)

$$V\rho = \boldsymbol{m}$$

$\boldsymbol{m} = 0.5 \times 536$ (substitute values and calculate)

$$\boldsymbol{m} = 268$$

Finally, calculate the Volume (V) of Lithium

Given: Density (ρ) = 536 and Mass (m) = 268

$$\rho = \frac{m}{\boldsymbol{V}}$$

$\boldsymbol{V}\rho = m$ (as above)

$$\frac{\boldsymbol{V}\cancel{\rho}}{\cancel{\rho}} = \frac{m}{\rho}$$

$$\boldsymbol{V} = \frac{m}{\rho}$$

$\boldsymbol{V} = \frac{268}{536}$ = 0.5

Many other equations use exactly the same process of rearranging

For example: $\vec{a} = \frac{\Delta\vec{v}}{t}$, $\vec{F} = m\vec{a}$, $P = \frac{F}{A}$, $f = \frac{1}{T}$, $v = \frac{d}{t}$, $M = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$ and $n_m = \frac{C_A}{C_m}$

Question 18:

a. **Using $\boldsymbol{v = f\lambda}$**

i. Given f = 3 and λ = 9, calculate v

ii. Given v = 40 and λ = 5, rearrange the equation to calculate f

iii. Given v = 60 and f = 3, rearrange the equation to calculate λ

b. **Using $\boldsymbol{\vec{F} = m\vec{a}}$**

i. Given m = 12 and $\vec{a}$ = 4, calculate $\vec{F}$

ii. Given $\vec{F}$= 81 and $\vec{a}$ =3, calculate m

iii. Given $\vec{F}$= 48 and calculate m=12, calculate $\vec{a}$

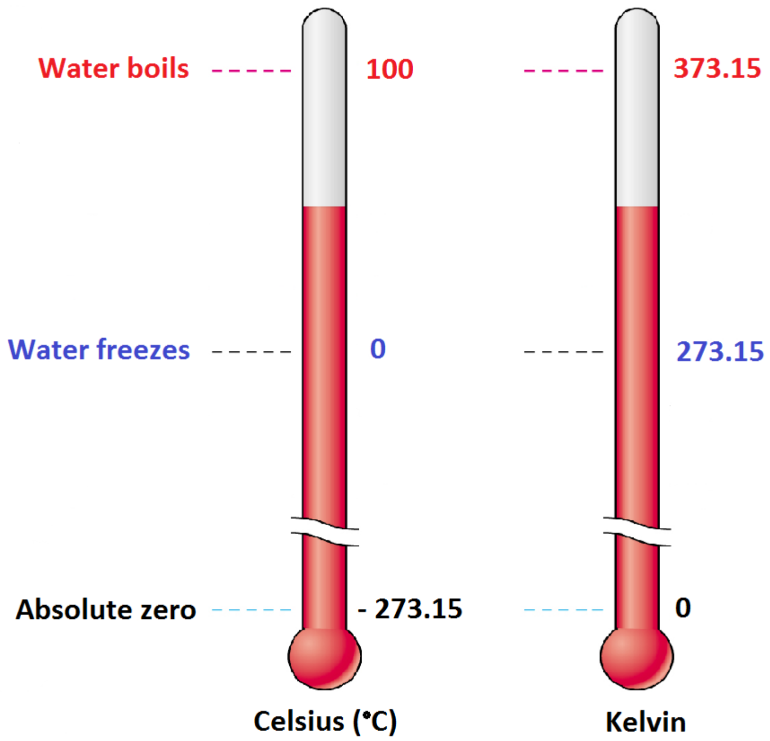
c. **Using $\boldsymbol{\vec{F}\Delta t = \Delta\vec{p}}$**

i. Given Δt = 5 and $\Delta\vec{p}$ = 50, rearrange the equation to calculate $\vec{F}$

ii. Given $\vec{F}$ = 5 and $\Delta\vec{p}$ = 30, rearrange the equation to calculate Δt

17. Temperature Conversions

Temperature is measured on different temperature scales. Celsius and Kelvin are the two most important temperature scales for scientific measurement. The Celsius scale, formerly known as the centigrade scale, is the scale at which water freezes at 0°C and boils at 100°C. There are 100 Celsius degrees between the two defined points: the boiling point and the freezing point. The Kelvin scale is an absolute temperature scale for which the zero point is absolute zero: the theoretical temperature at which molecules have the lowest energy. The Kelvin scale is related to the Celsius scale; a Celsius degree is the same size as a Kelvin and both scales have 100 divisions between the boiling and freezing points of water. The difference between the two scales is the zero point.



At times you may be required to convert between the two scales. The relationship between the two scales, represented mathematically, is:

°C = K – 273.15

K = °C + 273.15

Remember, the Kelvin scale does not use the degree (°) symbol.

Example:

The weather report says the temperature is 33 °C. What is this temperature on the Kelvin scale?

K = °C + 273.15

K = 33 + 273.15

= 306.15 K

The temperature of a solution is measured as 352 K. What is this temperature on the Celsius scale?

°C = K – 273.15

°C = 352 – 273.15

= 78.85 °C

Question 19:

a) Convert the following from Kelvins to degrees Celsius:

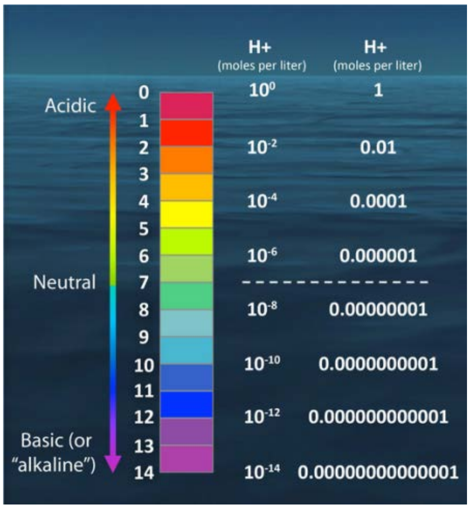
(i) 562 K
(ii) 184.89 K
(iii) 297 K
(iv) 342.65 K

b) Convert the following from degrees Celsius to Kelvin:

(v) 65 °C
(vi) 23.65 °C
(vii) 79.12 °C
(viii) 15 °C

18. Answers

1. A) 1 B) 22 C) 10 D) 12 E) ÷
2. 1) 0.6kg / 600, 000kg 2) 0.004264kL / 4264mL 3) 6.7m / 0.0067km
3. A) 301.44cm^3 B) 100.48cm^3
4. I) 7% II) 46% III) 9.2% IV) $16.5/kg
5. A) $\frac{5}{12}$ B) $\frac{14}{39}$ C) $\frac{14}{54}$ D) $\frac{15}{4}$ or $1\frac{1}{14}$ E) $\frac{108}{175}$ F) $-\frac{1}{2}$ G) $\frac{63}{8}$ H) 4 I) 22 out of 25 J) ½ K) 1/200 Moles
6. A) $\frac{13}{20}$ B) $\frac{27}{50}$ C) $2\frac{2}{3}$ D) $3\frac{7}{50}$ E) 8
7. A) 0.739 B) 0.069 C) 56.667 D) 5.8
8. I) 240min II) 21 Eggs III) 15 min
9. I) 4 molecules H_2SO_4 II) 8 molecules Al_2O_3 III) 18 molecules of $KClO_3$
10. A) 8 B) 81 C) 1/8 D)16 E)-27 F)1/16 G) 0 H) 1/100 I) 1/8 J) 1/16 K) -1 L) ¼ M) -1/4 N) 1/16 O) 64/27 P) -1/8
11. A) $5^6 + 5^2$ B) x^7 C) t^3 D) 5^{12} E) $2^4 \times 3^2$ F) 3^{-3} G) $\frac{3x^5}{y^2}$ H) $\frac{1}{\sqrt{a}}$ I) 2 J) -2 K) 3 L) 5
    M) $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = \frac{16a^2b^3}{3a^3b} \times \frac{9a^3b^5}{8b^2a} = \frac{16a^{-1}b^2}{3} \times \frac{9a^2b^3}{8} = \frac{144a^1b^5}{24} = 6ab^5$
    A) 6×10^6 B) 2×10^4 C) 5.6×10^5 D) 3×10^2 E) 1 F) 77.44×10^{-2}
12. A) 4.5×10^2 B) 9.0×10^7 C) 3.5×10^0 D) 9.75×10^{-2} E) 375 F) 39.7 G) 0.1875 H) 0.00875
13. A) 3.54×10^2 (3 sig) B) 1.8932×10^4 (5 sig) C) 4.4×10^{-3} (2 sig) D) 5.06×10^{-4} (3 sig) E) 9.6×10^2 (2 sig) F) 3.0×10^4 (1 sig) G) 1.50900×10^2 (6 sig)
14. A) 3 B) 8 C) 7 D) 12 E) pH=-log$_{10}$[H$^+$]
15. A) log12 B) log27 C) 1 D) 2 E) 5×10^{-4}
16. A) 15 B) 16 C) 9 D) 14
17. A) 7 B) 162 C) $20\frac{2}{3}$ $\left(or\ \frac{62}{3}\right)$ D) 13 E) 154
18. A I) 27 II) 8 III) 20 B I) 48 II) 27 III) 4 C I) 10 II)6
19. A I) 288.85℃ II) -88.26℃ III) 23.85℃ IV) 69.5℃ B I) 338.15K II) 296.8K III) 352.27K IV) 288.15K



For more information regarding basic math resources please refer to your LearnJCU. There are more resources that you can access via The Learning Centre website https://www.jcu.edu.au/students/learning-centre There are resources and information on short courses, time management, assignments, exam preparations etc.

References:

Brown, T. L., LeMay, H. E., & Bursten, B. E. (2006). *Chemistry the Central Science* (10th ed.). Upper Saddle River, NJ: Pearson.

Seager, S. L., & Slabaugh, M. R. (2011). *Introductory Chemistry for Today* (7th ed.). Belmont, CA: Brooks/Cole.

Tro, N. J. (2011). *Introductory Chemistry* (4th ed.). Glenview, IL: Pearson.