Learning, Teaching and Student Engagement

JAMES COOK UNIVERSITY AUSTRALIA

Cairns Singapore Townsville

Maths Module 9

Mathematics for Introductory Chemistry

This module covers concepts such as:

- Measurement units
- Scientific notation
- Significant figures
- percentages and ratios
- Solving equations

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Module 9

Mathematics for Introductory Chemistry

1. Measurement Units
2. Area
3. Scientific Notation
4. Significant Figures
5. Calculating Percentages
6. Ratios
7. Solving Equations
8. Temperature Conversions

1. Measurement Units

Measurements consist of two parts – the number and the identifying unit.

Number → **50 kg** ← Identifying unit

In scientific measurements, units derived from the metric system are the preferred units. The metric system is a decimal system in which larger and smaller units are related by factors of 10.

Table 1. Common Prefixes of the Metric System

Prefix	Abbreviation	Relationship to basic unit	Exponential Relationship to basic unit
mega-	M	1 000 000 x basic unit	10^{6} x basic unit
kilo-	k	1000 x basic unit	10^{3} x basic unit
deci-	d	1/10 x basic unit **or** 0.1 x basic unit	10^{-1} x basic unit
centi-	c	1/100 x basic unit **or** 0.01 x basic unit	10^{-2} x basic unit
milli-	m	1/1000 x basic unit **or** 0.001 x basic unit	10^{-3} x basic unit
micro-	µ	1/1 000 000 x basic unit **or** 0.000001 x basic unit	10^{-6} x basic unit
nano-	n	1/1 000 000 000 x basic unit **or** 0.000000001 x basic unit	10^{-9} x basic unit

Complete this table:

Base Unit	Larger Unit	Smaller Unit
1 metre	1 kilometre = 1000 meters	100 centimetres = 1 meter 1000 millimetres = 1 meter
1 gram	1 kilogram = 1000 grams	1000 milligrams = 1 gram 1 000 000 micrograms = 1 gram
1 litre		

Example:

Convert 0.15 g to kilograms and milligrams	Convert 5234 mL to litres
Because 1 kg = 1000 g, 0.15 g can be converted to kilograms as shown: $\mathbf{0.15\ g\ x\ \frac{1\ kg}{1000\ g} = 0.00015\ kg}$ Also, because 1 g = 1000 mg, 0.15 g can be converted to milligrams as shown: $\mathbf{0.15\ \not{g}\ x\ \frac{1000\ mg}{1\ \not{g}} = 150\ mg}$	Because 1 L = 1000 mL, 5234 mL can be converted to litres as shown: $\mathbf{5234 mL\ x\ \frac{1\ L}{1000\ mL} = 5.234\ L}$

Your turn:

1 a) Convert 600 g to kilograms and milligrams

b) Convert 4.264 L to kilolitres and millilitres

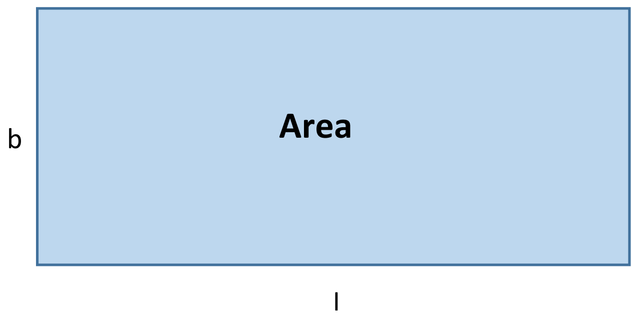
c) Convert 670 cm to metres and kilometres

*Refer to Maths Module 7: Geometry (p. 13-15) for further assistance with unit conversions

2. Area

Area is a measure of the amount of space a two dimensional shape takes up; the space that is enclosed by its boundary. Area is measured in squared units and is calculated by multiplying the length by the breadth.

area (A) = l x b



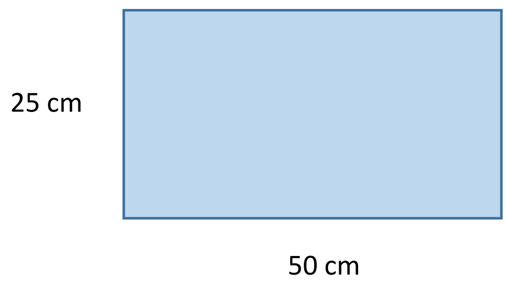
Example:

Calculate the area of a rectangle that has sides of 50 cm and 25 cm. Express your answer in square centimetres: cm^2 represents square centimetres.

A = l x b

= 50 cm x 25 cm

= 1250 cm^2



Your turn:

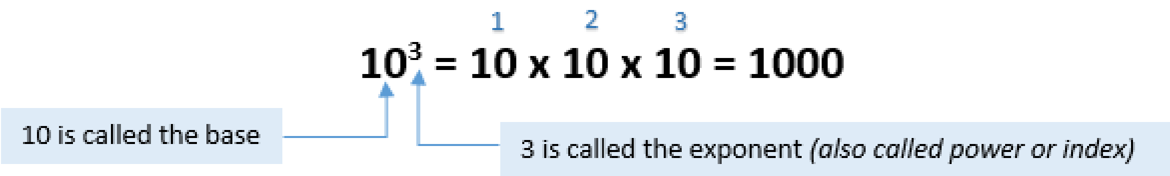
2 a) Calculate the area of a rectangle that has sides of 12 m and 6 m. Express your answer in square metres: m^2 represents square metres.

b) Calculate the area of a rectangle that has sides of 152 cm and 620 cm.

**Refer to Maths Module 7: Geometry (p. 7-8) for further assistance with calculating area*

3. Scientific Notation

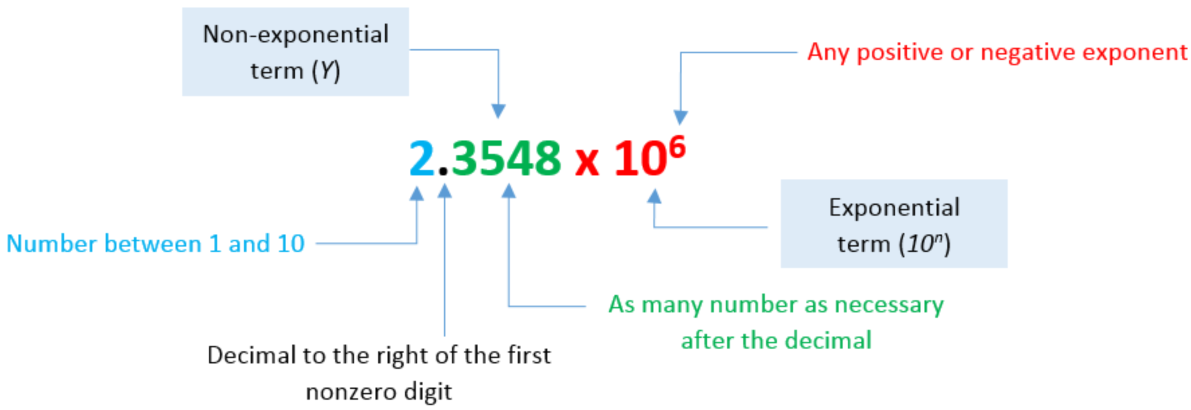
Numbers as multiples or fractions of ten	Number	Number as a power of ten
10 x 10 x 10 x 10	10 000	10^4
10 x 10 x 10	1000	10^3
10 x 10	100	10^2
10	10	10^1
10 x 1/10	1	10^0
1/10	0.1	10^{-1}
1/100	0.01	10^{-2}
1/1000	0.001	10^{-3}
1/10 000	0.0001	10^{-4}



Scientific notation is a convenient method of representing and working with very large and very small numbers.

In scientific notation, numbers are represented as the product of a non-exponential term and an exponential term in the general form $Y \times 10^n$.

The non-exponential term (Y) is a number between 1 and 10, but not equal to 10, written with a decimal to the right of the first nonzero digit in the number. The exponential term is a 10 raised to a whole number exponent (n) that may be positive or negative. The value of n is the number of places the decimal must be moved from the position in Y to be at the original position when the number is written without using scientific notation.



If n is positive, shift the decimal point that many places to the right.

If n is negative, shift the decimal point that many places to the left.

Example:

The following numbers are written using scientific notation. Write them in original form, without using scientific notation.

a) 4.81 x 10^6^

The exponent +6 indicates that the original position of the decimal is located 6 places to the right of the decimal position in 4.81. Zeros are added to provide for this change:

$$4.81 \times 10^{6} = 4\ 8\ 1\ 0\ 0\ 0\ 0\ . = 4\ 810\ 000$$

1 2 3 4 5 6

b) 6.234 x 10^–7^

The exponent –7 indicates that the original position of the decimal is 7 places to the left of the decimal position in 6.234. Again, zeros are added as required:

$$6.234 \times 10^{-7} = .\ 0\ 0\ 0\ 0\ 0\ 0\ 6\ 2\ 3\ 4 = 0.0000006234$$

7 6 5 4 3 2 1

Write the following numbers using scientific notation.

c) 89 530 000

The non-exponent term must be a number between 1 and 10, written with a decimal to the right of the first nonzero digit:

8.953

The original decimal position is 7 places to the right of this position so the exponent must be +7:

$$8\ 9\ 5\ 3\ 0\ 0\ 0\ 0\ . = 8.953 \times 10^{7}$$

1 2 3 4 5 6 7

d) 0.00000000524

The non-exponent term must be a number between 1 and 10, written with a decimal to the right of the first nonzero digit:

5.24

The original decimal position is 9 places to the left of this position so the exponent must be –9:

$$.0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 5\ 2\ 4 = 5.24 \times 10^{-9}$$

Your turn:

3 a) Write the following numbers using scientific notation:

(i) 780 000

(ii) 5432.66

(iii) 0.000798

(iv) 0.00000065

b) The following numbers are written using scientific notation. Write them in standard form:

(i) 5.269×10^{7}

(ii) 3.66×10^{-6}

(iii) 7.256×10^{3}

(iv) 4.21×10^{-4}

*Refer to Maths Module 4: Powers, Roots and Logarithms (p.5) for further assistance with scientific notation

4. Significant Figures

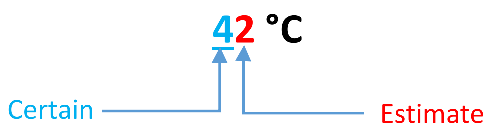
Significant figures are the numbers in a measurement that are known with some degree of confidence. As the precision of a measurement increases, so does the number of significant figures.

Every measurement contains uncertainty that is representative of the device used to take the measurement. These uncertainties are characterised by the number of digits used to record the measurement; the number of **significant figures** used. Consider the thermometers below:

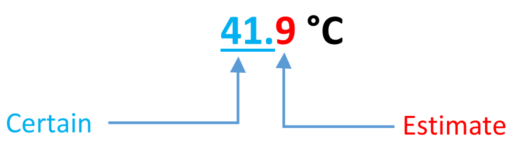
A

B

In (A), the temperature is measured with a thermometer with markings every 10 °C. We can see that the temperature is greater than 40 °C, but less than 50 °C. The temperature is recorded by writing the number that is known with certainty to be correct and writing an estimate for the uncertain number:



In (B), the temperature is measured with a thermometer with markings every 1°C. We can see that the temperature is at least 41 °C, but not quite 42 °C. Once again, the certain numbers are recorded and an estimate is made for the uncertain part:



When measurements are recorded this way, the numbers representing the certain measurement plus the one number representing the uncertain measurement are called **significant figures**. The first measurement of 42 °C contains two significant figures, while the second measurement of 41.9 °C contains three significant figures.

To determine the number of significant figures in a number, follow these conventions:	
1. All nonzero digits are significant	529 has three significant figures 1.562 has four significant figures
2. Zeros between two nonzero digits are significant	3063 has four significant figures 1.0302 has five significant figures
3. Leading zeros (zeros to the left of the first nonzero digit) are not significant	0.000078 has two significant figures (this is more easily seen if written as 7.8×10^{-5})
4. Trailing zeros (zeros to the right of a nonzero number) that fall after a decimal point are significant	5.10 has three significant figures 0.400 has three significant figures
5. Trailing zeros that fall before a decimal point are significant	18000.1 has six significant figures 500.0 has four significant figures
6. Trailing zeros at the end of a number, but before an implied decimal point, are ambiguous and should be avoided by using scientific notation indicating the exact number of significant figures	It is unclear whether 180 has two or three significant figures. 1.8×10^2 indicates two significant figures while 1.80×10^2 indicates three significant figures

Your turn:

4 a) How many significant figures are in each number?

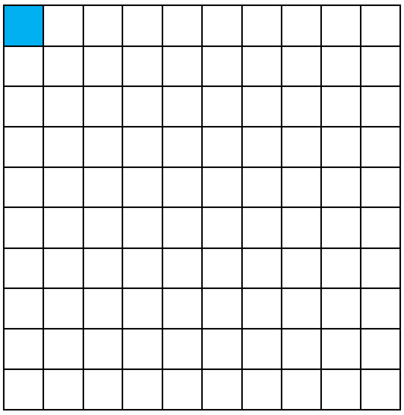
(i) **0.00065**
(ii) **6.0902**
(iii) **45.00**
(iv) **8.79×10^6**
(v) **2300.100**

**Refer to Maths Module 4: Powers, Roots and Logarithms (p.5-7) for further assistance with significant figures*

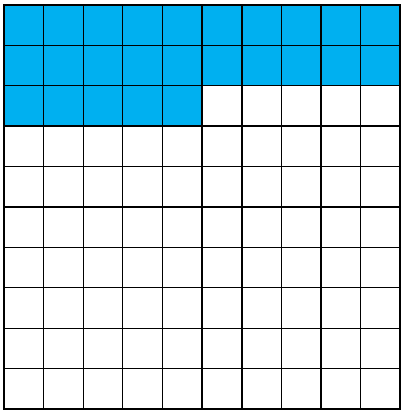
5. Calculating Percentages

A percentage is a number expressed as a fraction of 100. The word *percent* actually means per one hundred.

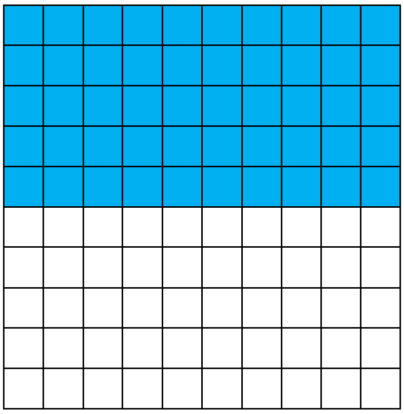
The word *percent* means per 100. It is the number of units in a group of 100 units.



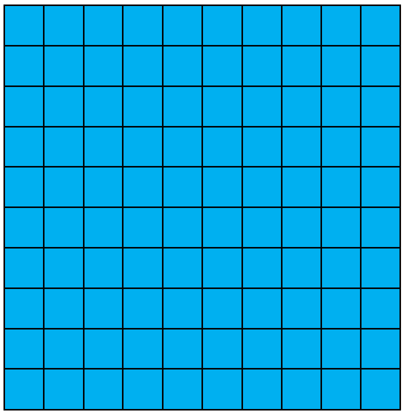
One percent (**1 %**) means 1 per 100 or $\frac{1}{100}$



25 % means 25 per 100 or $\frac{25}{100}$



50 % means 50 per 100 or $\frac{50}{100}$



100 % means 100 per 100 or $\frac{100}{100}$

Items/measurements are rarely found in groups of exactly 100, so we can calculate the number of units that would be in the group if it did contain the exactly 100 units – the percentage.

$$Percent\ (\%) = \frac{number\ of\ units}{total\ number\ of\ items\ in\ the\ group} \times 100$$

Example:

JCU has 2154 female and 1978 male students enrolled. What percentage of the students is female?

The total number of students is 4132, of which 2154 are female.

$$\% \text{ female} = \frac{number\ of\ females}{total\ number\ of\ students} \times 100$$

$$= \frac{2154}{4132} \times 100$$

$$= 52.13\ \%$$

When John is exercising his heart rate rises to 180 bpm. His resting heart rate is 70 % of this. What is his resting heart rate?

$$70\ \% = \frac{resting\ heart\ rate}{180} \times 100$$

Rearrange:

Start with $70\ \% = \frac{resting\ heart\ rate}{180} \times 100$

Divide both sides by 100 $\frac{70}{100} = \frac{resting\ heart\ rate}{180}$

Multiply both sides by 180 $\frac{70}{100} \times 180 = resting\ heart\ rate$

$$\text{Resting heart rate} = \frac{70}{100} \times 180$$

$$= 126 \text{ bpm}$$

Your turn:

5 a) Calculate the following:

(i) Sally bought a television that was advertised for $467.80. She received a discount of $32.75. What percentage discount did she receive?

(ii) 50 kg of olives yields 23 kg of olive oil. What percentage of the olives' mass was lost during the extraction process?

(iii) The recommended daily energy intake for women is 8700 KJ. What percentage over the recommended intake is a women consuming if her energy intake is 9500 KJ?

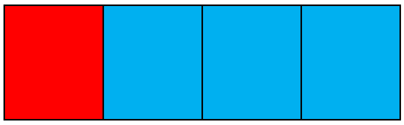
(iv) An advertisement the fruit and vegetable shop states that there is 25 % off everything. Bananas are normally $22/Kg, what is the new per kilo price?

**Refer to Maths Module 3: Ratio, Proportion and Percent (p.11-13) for further assistance with percentages*

6. Ratios

A ratio is a comparison of the size of one number to the size of another number. A ratio represents for every determined amount of one thing, how much there is of another thing.

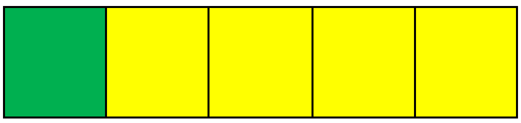
Ratios use the symbol **:** to separate the quantities being compared. For example, 1:3 means 1 unit to 3 units.



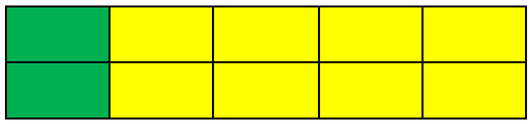
- There is 1 red square to 3 blue squares
- 1:3
- 1 to 3

Ratios can be expressed as fractions but you can see from the above diagram that 1:3 is not the same as $\frac{1}{3}$

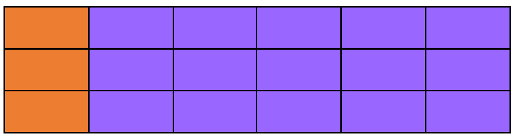
A ratio can be scaled up:



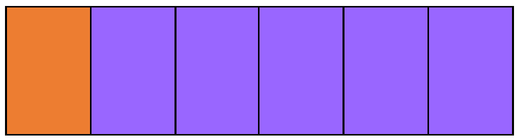
1:4 = 2:8



Or scaled down:



3:15 = 1:5



1:5 is the same as 2:10 is the same as 3:15 is the same as 4:20 and so on

Example:

A pancake recipe requires flour and milk to be mixed to a ratio of 1:3. This means one part flour to 3 parts milk. No matter what device I use to measure, the ratio must stay the same.

So if I add 200 mL of flour, I add 200 mL x 3 = 600 mL of milk

If I add 1 cup of flour, I add 3 cups of milk

If I add 50 grams of flour, I add 150 grams of milk

Your turn:

6 a) Calculate the following:

(i) Jane reads 25 pages in 30 minutes. How long does it take her to read 200 pages?

(ii) John uses 7 eggs to make 4 muffins. How many eggs does he need to make 12 muffins?

(iii) Jane is swimming laps at the local swimming pool. She swims 4 laps in 3 minutes. How long does it take her to swim 20 laps?

Ratios are used in **stoichiometry** – the study of mass relationships in chemical reactions. Consider the following equation:

$$2HCl_{(aq)} + CaO_{(s)} \rightarrow CaCl_{2(aq)} + H_2O_{(l)}$$

This equation is showing that 2 HCl molecules react with 1 CaO molecule to form 1 $CaCl_2$ molecule and 1 H_2O molecule.

$$2HCl_{(aq)} + CaO_{(s)} \rightarrow CaCl_{2(aq)} + H_2O_{(l)}$$

2 : 1 : 1 : 1

Example:

How many molecules of CaO would you need to react with 24 molecules of HCl?

$$2HCl_{(aq)} + CaO_{(s)} \rightarrow CaCl_{2(aq)} + H_2O_{(l)}$$

2 : 1 : 1 : 1

The reaction between HCl and CaO uses 2 HCl molecules for every 1 CaO molecule.

Ratio is 2 HCl : 1 CaO

24 HCl : 12 CaO

12 molecules of CaO is required.

Your turn:

6 b) Calculate the following:

(i) $H_2SO_{4(aq)} + 2LiOH_{(aq)} \rightarrow LiSO_{4(aq)} + 2H_2O_{(l)}$
How many molecules of H_2SO_4 are required to react with 8 molecules of LiOH?

(ii) $3MnO_{2(s)} + 4Al_{(s)} \rightarrow 2Al_2O_{3(s)} + 3Mn_{(s)}$
How many molecules of Al_2O_3 are produced if we have 12 molecules of MnO_2?

(iii) $2KClO_{3(s)} \rightarrow 2KCl_{(s)} + 3O_{2(g)}$
How many molecules of $KClO_3$ are required to produce 27 molecules of O_2?

**Refer to Maths Module 3: Ratio, Proportion and Percent (p.3-4) for further assistance with ratios*

7. Solving Equations

An equation states that two quantities are equal – it will have an '=' sign. An equation shows that the left hand and right hand sides of the equals sign are equivalent – they balance. The equation may contain an unknown quantity that we wish to find.

In this equation, the unknown quantity is x

$2x + 10 = 50$

This equation states:

2 multiplied by some number and then added to 10 will equal 50

To solve the equation means to find all the values of the unknown quantity, to make the left side equal to the right side. This value is called a solution. In the example above, the solution is $x = 20$. When 20 is substituted into the equation for x, both sides of the equation equal 50.

To solve an equation we are often required to rearrange the equation – the goal being to end up with:

x = something

We want to move everything, except 'x' (or whatever the variable is named) over to the right hand side

Remember, what we do to one side of the equation, we must do to the other!

In the example above:

Start with	**$2x + 10 = 50$**
Minus 10 from both sides	**$2x = 40$**
Divide both sides by 2	**$x = 20$**

* Once the equation has been solved it is important to check the solution. Substitute the solution back into the equation to make sure it is correct.

$2(20) + 10 = 50$

$40 + 10 = 50$ ✓✓

Example:

Solve 3x – 3 = 21

Start with	**$3x - 3 = 21$**
Add 3 to both sides	**$3x = 24$**
Divide both sides by 3	**$x = 8$**

Your turn:

7 a) Solve the following:

(i) 7a + 9 = 51
(ii) 4y – 8 = 0
(iii) 2 + 6x – 4 = 28
(iv) 87 = 7 + 4x

A formula is an equation which shows the relationship between variables. Unlike an equation, a formula will have more than one variable.

In this equation, x and y are variables	**$2y = 3x - 6$**	The left hand side equals the right hand side

If we were to solve this formula for x, our goal would be to end up with x = something. We rearrange the formula to achieve this:

Start with	**$2y = 3x - 6$**
Add 6 to both sides	**$2y + 6 = 3x$**
Divide both sides by 3	$\frac{2y+6}{3} = x$

If we were to solve this formula for y, our goal would be to end up with x = something. We rearrange the formula to achieve this:

Start with	**$2y = 3x - 6$**
Divide both sides by 2	$y = \frac{3x-6}{2}$

Your turn:

7 b) Solve the following for x:

(i) 7a = 3x + 9
(ii) 4y + 3x = 13
(iii) 2z + 6x – 4 = 28

In chemistry formulas are often used to calculate quantities.

$$density = \frac{mass}{volume}$$

$$\rho = \frac{m}{V}$$

Density is the value obtained by dividing the mass of a sample by the volume of that sample

Example:

The mass of an aluminum sample is 21.6 g. The volume of the same sample is 8 cm^3. What is the density of aluminum metal?

$$\rho = \frac{m}{V}$$

The formula shows the relationship between variables and can be used to solve the problem

Substitute the given values into the formula

$$\rho = \frac{21.6}{8}$$

ρ = 2.7 g/cm^3

The density of aluminum metal has been determined to be 2.7 g/cm^3. Calculate the mass of the sample if the volume is 50 cm^3.

$$\rho = \frac{m}{V}$$

Rearrange the equation to solve for m (mass)

$$m = \rho v$$

Substitute the given values into the formula

m = 2.7 x 50

m = 135 g

Your turn:

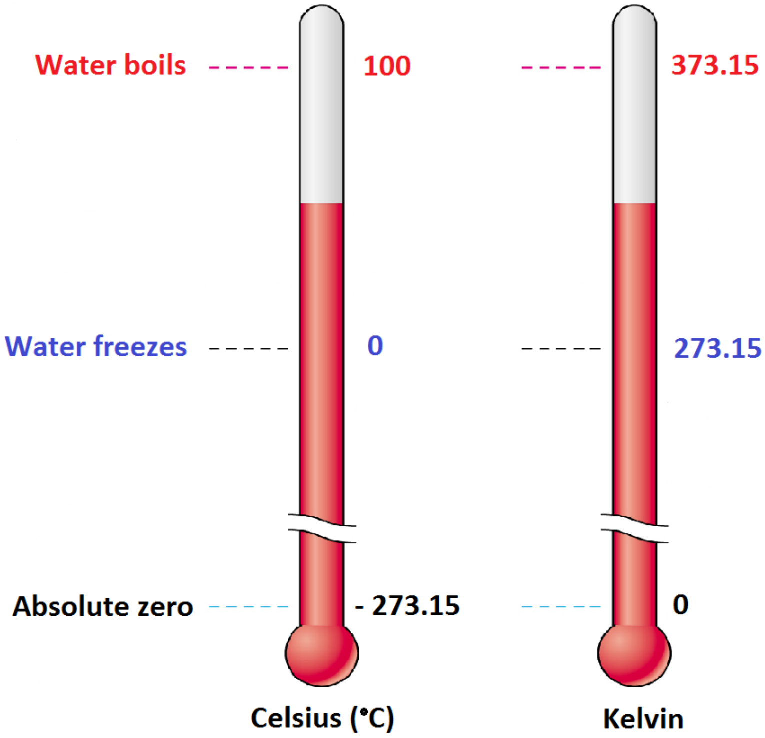
7 c) Solve the following:

(i) The density of iron metal has been determined to be 7.2 g/cm^3. Calculate the volume of the sample if the mass is 648 g.

(ii) Dilution calculations can use the relationship $C_1V_1 = C_2V_2$
If C_1 = 5 mol/L, V_1 = 0.3 L and C_2 = 2.7 mol/L what is the value of V_2?

8. Temperature Conversions

Temperature is measured on different temperature scales. Celsius and Kelvin are the two most important temperature scales for scientific measurement. The Celsius scale, formerly known as the centigrade scale, is the scale at which water freezes at 0°C and boils at 100°C. There are 100 Celsius degrees between the two defined points: the boiling point and the freezing point. The Kelvin scale is an absolute temperature scale for which the zero point is absolute zero: the theoretical temperature at which molecules have the lowest energy. The Kelvin scale is related to the Celsius scale; a Celsius degree is the same size as a Kelvin and both scales have 100 divisions between the boiling and freezing points of water. The difference between the two scales is the zero point.



At times you may be required to convert between the two scales. The relationship between the two scales, represented mathematically, is:

°C = K – 273.15

K = °C + 273.15

Remember, the Kelvin scale does not use the degree (°) symbol.

Example:

The weather report says the temperature is 33 °C. What is this temperature on the Kelvin scale?

K = °C + 273.15

K = 33 + 273.15

= 306.15 K

The temperature of a solution is measured as 352 K. What is this temperature on the Celsius scale?

°C = K – 273.15

°C = 352 – 273.15

= 78.85 °C

Your turn:

8 a) Convert the following from Kelvins to degrees Celsius:

(i) 562 K
(ii) 184.89 K
(iii) 297 K
(iv) 342.65 K

8 b) Convert the following from degrees Celsius to Kelvin:

(v) 65 °C
(vi) 23.65 °C
(vii) 79.12 °C
(viii) 15 °C

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