James Cook University Australia

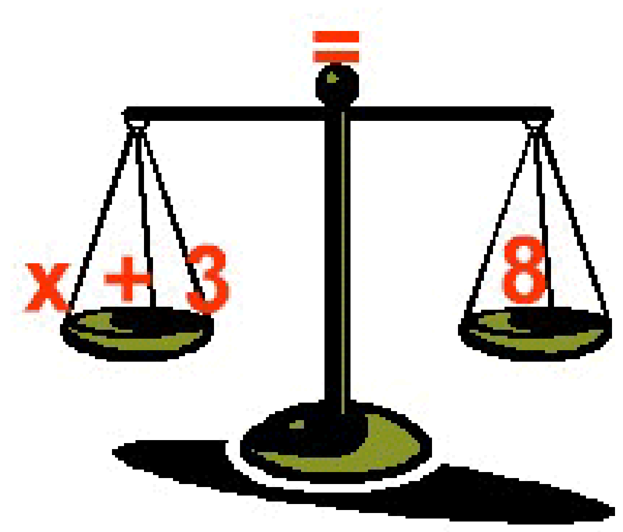
Maths Refresher

Solving Equations

Solving Equations

Learning intentions ….

- Transposing formulae
- Types of equations
- Solving linear equations



What’s the speed limit?

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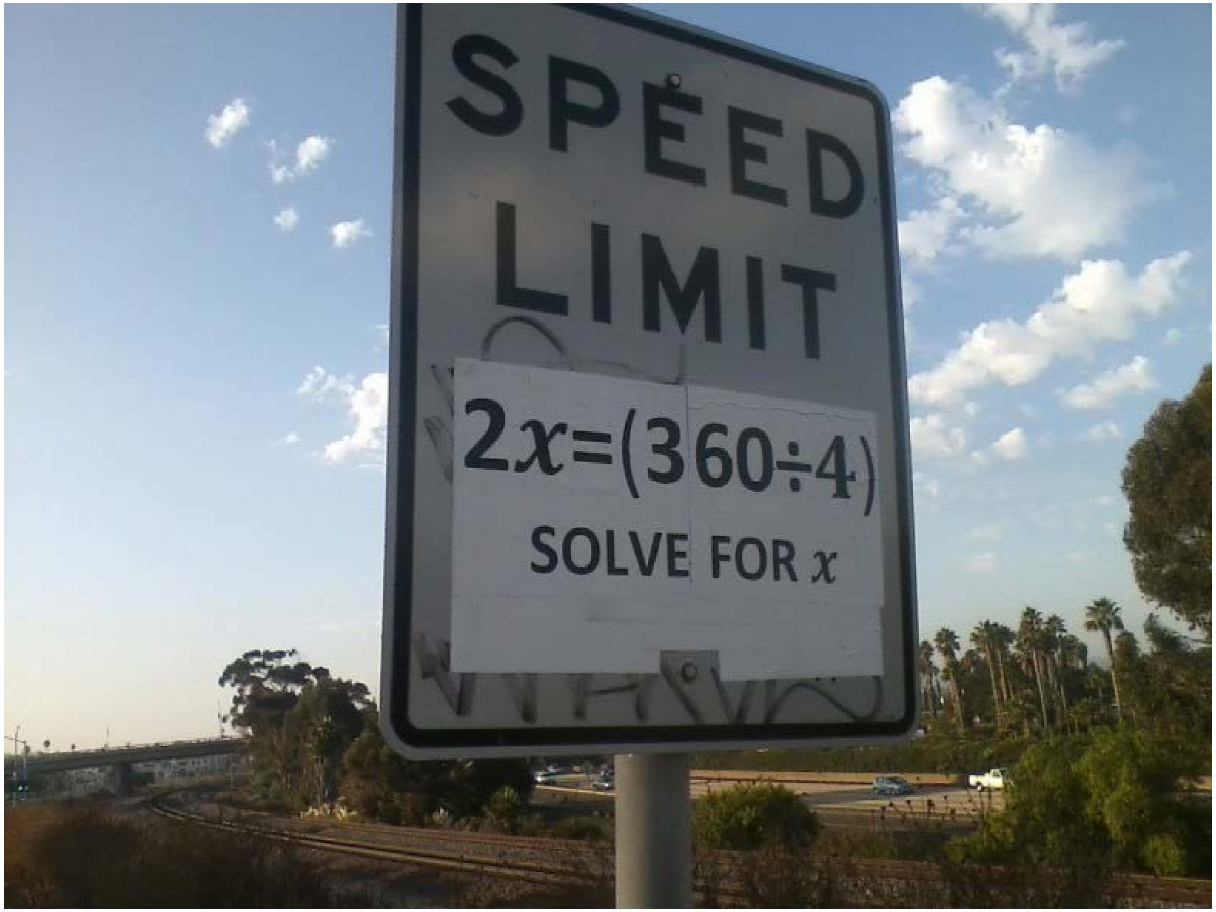

What’s the time?

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Expression versus equation

Expression	Equation
$2x + 4$	$2x + 4 = 8$
$Let\ x = 1$	$2x + 4 - 4 = 8 - 4$
$2 \times 1 + 4$	$2x = 4$
$= 2 + 4$	$2x \div 2 = 4 \div 2$
$= 6$	$x = 2$
OR	
$Let\ x = 5$	
$2 \times 5 + 4$	
$= 10 + 4$	
$= 14$	

Solving equations

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- An **equation** states that two quantities are equal, and will always contain an **unknown quantity** we wish to find. For example, in the equation $5x + 10 = 20$, the unknown quantity is x.
- To solve an equation means to find all values of the unknown quantity so that they can be substituted to make the left side equal the right side.
- Each such value is called a **solution**, or alternatively a **root** of the equation. In the example above, the solution is $x = 2$ because when 2 is substituted, both the left side and the right side equal 20; the sides balance.
- The value $x = 2$ is said to **satisfy** the equation

(Croft & Davison, 2010, p. 109)

Transposing formulae

Sometimes, in order to solve an equation, it is necessary to **transpose** or **rearrange** the formula.

The essential rule to remember is that whatever you do to one side, you must also do to the whole of the other side.

For example, you may:

- Add the same quantity to both sides
- Subtract the same quantity from both sides
- Multiply or divide both sides by the same quantity
- Perform operations on both sides such as ‘square both sides’, ‘square root both sides’ , etc.

(Croft & Davison, 2010, p. 103)

Transposing: Example 1

The circumference C of a circle is given by the formula C = $2\pi r$. Transpose this to make *r* the subject.

The intention is to obtain r by itself on the LHS.

Starting with C = $2\pi r$, *we can then divide both sides by* 2π

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$ *On the RHS, we can cancel the common factor of* 2π

$$\frac{C}{2\pi} = r$$

$$\therefore r = \frac{C}{2\pi}$$

Transposing: Example 2

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Transpose the formula $y = 3(x + 7)$ for x

$\frac{y}{3} = \frac{3(x+7)}{3}$ *divide both sides by 3*

$\frac{y}{3} = x + 7$ *then, subtract 7 from both sides*

$\frac{y}{3} - 7 = x + 7 - 7$

$\frac{y}{3} - 7 = x$ *transpose so that x is on the LHS*

$x = \frac{y}{3} - 7$

Your turn …

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Transpose each of the following formulae to make x the subject:

(a) $y = 3x$

(b) $y = \frac{1}{x}$

(c) $y = 7x – 5$

(d) $y = \frac{1}{2}x – 7$

(e) $y = \frac{1}{2x}$

(f) $y = 13(x-2)$

(g) $y = x(1 + \frac{1}{x})$

(h) $y = a + t\,(x - 3)$

Answers…

Transpose each of the following formulae to make x the subject:

(a) $y = 3x$ $x = \frac{y}{3}$

(b) $y = \frac{1}{x}$ $x = \frac{1}{y}$

(c) $y = 7x - 5$ $x = \frac{5+y}{7}$

(d) $y = \frac{1}{2}x - 7$ $x = 2y + 14$

(e) $y = \frac{1}{2x}$ $x = \frac{1}{2y}$

(f) $y = 13(x-2)$ $x = \frac{y+26}{13}$

(g) $y = x(1 + \frac{1}{x})$ $y = x + 1$ and so $x = y - 1$

(h) $y = a + t(x - 3)$ $y = a + tx - 3t$ and so

$tx = y + 3t - a$ and then

$x = \frac{y+3t-a}{t}$

There are different types of equations that you might encounter and need to solve:

Type of Equation	Key features	Example
Linear	There is one variable, with or without a coefficient, and a constant. They take the form: $ax + b = 0$	$2x + 4$ $x - 17$ $3x - 12.4$
Simultaneous	Equations that contain more than one unknown (e.g. x and y), and usually **two** equations (x and y will satisfy both equations simultaneously)	$x + 2y = 14$ and $3x + y = 17$ $x=4$, $y=5$
Quadratic (numbers raised to the second power)	Is where the independent variable is raised to the second power. Takes the form: $y = ax^2 + bx + c$ $a\ b\ and\ c$ are constants (a does not equal zero) x is the independent variable and y is the dependent variable	$y = x^2 + 8x + 15$ (where $a = 1$, $b=8$, $c=15$) $y = x^2 - 3x - 2$ (where $a = 1$, $b=-3$, $c=-2$ $y = 3x^2$ (where $a=3$, $b=0$, $c=0$)

Solving linear equations

- The following practise examples will focus on solving linear equations that involve various operations.
- Let us look at an example:

$$\mathbf{2}\boldsymbol{x}+\mathbf{6}=\mathbf{14}$$

In words: what number can we double then add six so we have a total of fourteen?

Apply the following two principles to solve the equation:

- Work towards the variable and “what we do to one side we must do to the other”
- So let’s solve the problem …

Solving linear equations

Step 1: $2x + 6 = 14$

The constant, 6, is our first target.

If we take 6 from both sides, we create the following equation:

$2x + 6 - 6 = 14 - 6$ (The opposite of +6 $is\ -6$)

$2x = 8$ $(+6 - 6\ =\ 0)$

Solving linear equations

Step 2: $2x = 8$

The only number left on the same side as the variable is the coefficient, 2. It is our second target.

If we divide both sides by two, we create the following equation (note: between the 2 and the *x* is an invisible multiplication sign):

$$\frac{2x}{2} = \frac{8}{2}$$

(The opposite of **×2** is ÷2 or /2)

$$x = 4$$

Solving linear equations

Step 3:

Check. $2x + 6 = 14$

If we substitute a 4 (x = 4) into the equation we have:

$2 \times 4 \ + 6 = 14$

$8 + 6 = 14$

$14 = 14$ (We are correct!)

Solving linear equations

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NOTE: To remove a **constant** or a **coefficient**, we perform the opposite operation on both sides.

Opposite of × is ÷

Opposite of + is –

Opposite of x^2 is $\sqrt{x}$

KHANACADEMY

Watch this short Khan Academy video for further explanation:
"Simple equations of the form ax=b"
https://www.khanacademy.org/math/algebra/solving-linear-equations-and-inequalities/equations_beginner/v/simple-equations

Solving linear equations:

Example linear equation 1:

Solve for J when $3J - 5 = 16$

$3J - 5 = 16$ (target 5 then 3)

$3J - 5 + 5 = 16 + 5$ (Opposite of –5 is +5)

$3J = 21$

$\frac{3J}{3} = \frac{21}{3}$ (Opposite of ×3 is ÷3)

$\therefore J = 7$

(Check: $3 \times 7 - 5 = 16$ ✓)

Solving linear equations:

Example linear equation 2:

$$\frac{3T}{12} - 7 = 6$$

(Target 7 then 12 then 3)

$$\frac{3T}{12} - 7 + 7 = 6 + 7$$
$$\frac{3T}{12} = 13$$

$$\frac{3T}{12} \times 12 = 13 \times 12$$
$$3T = 156$$
$$\frac{3T}{3} = \frac{156}{3}$$
$$T = 52$$

(Check: 3 × 52 ÷12 -7 = 6 ✓)

Your turn …

Practise equations:

(a) $5x + 9 = 44$

(b) $\frac{x}{9} + 12 = 30$

(c) $3y + 13 = 49$

(d) $4x - 10 = 42$

(e) $\frac{x}{11} + 16 = 30$

(f) $\frac{1}{x} = 5$

(g) $\frac{1}{x} = \frac{5}{2}$

(h) $\frac{1}{x+1} = \frac{5}{2}$

(i) $\frac{1}{x-1} = \frac{5}{2}$

(j) $\frac{1}{x} = \frac{1}{2x+1}$

(k) $\frac{1}{x+1} = \frac{1}{3x+2}$

(l) $\frac{3}{x+1} = \frac{2}{4x+1}$

Answers

(*a*) $x = 7$

(*b*) $x = 162$

(*c*) $y = 12$

(*d*) $x = 13$

(*e*) $x = 154$

(*f*) $x = \frac{1}{5}$

(*g*) $x = \frac{2}{5}$

(*h*) $x = -\frac{3}{5}$

(*i*) $x = 1\frac{2}{5}$

(*j*) $x = -1$

(*k*) $x = -\frac{1}{2}$

(*l*) $x = -\frac{1}{10}$

Remember powers & roots …

Can you solve this equation?

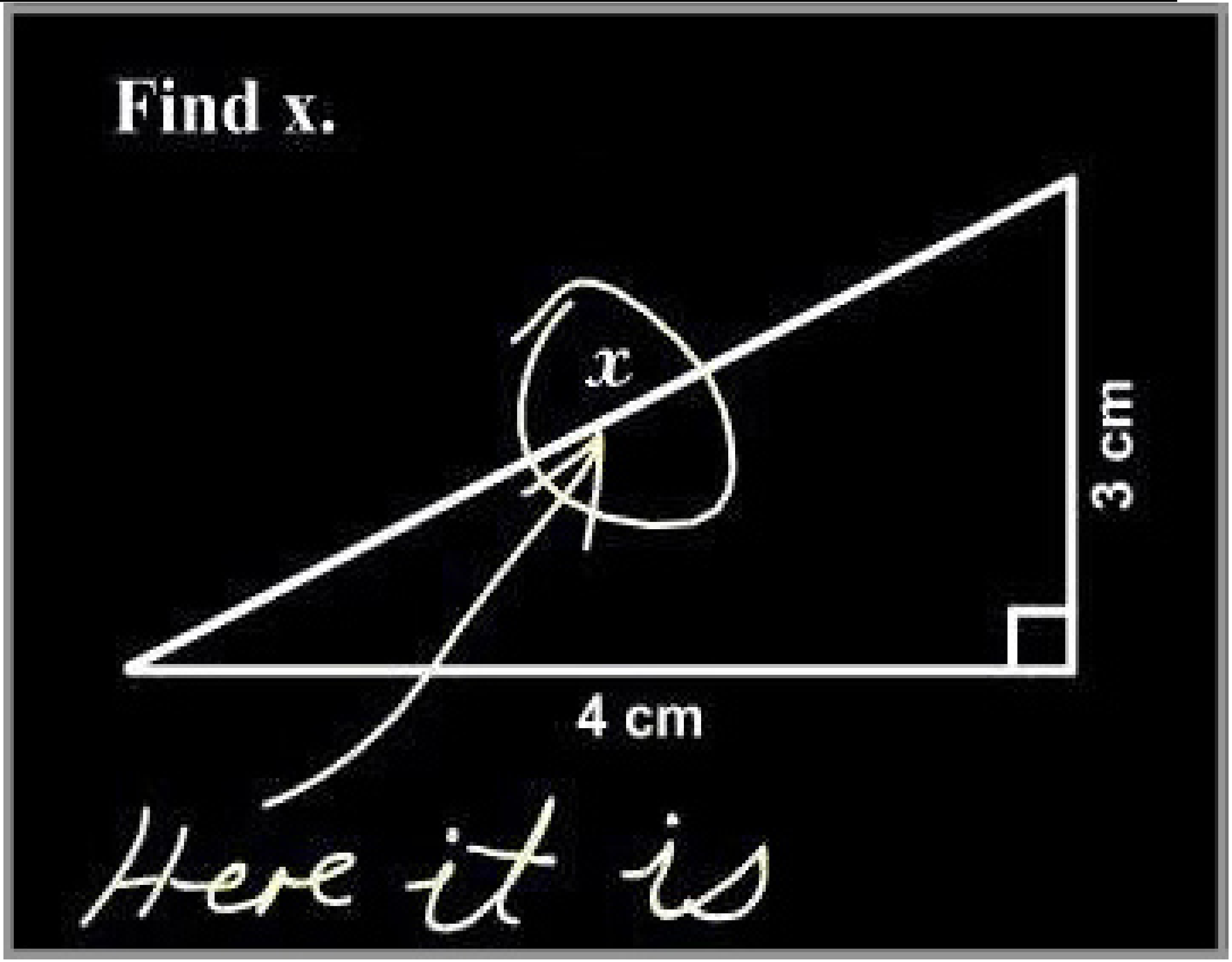
$x^2 = 16$

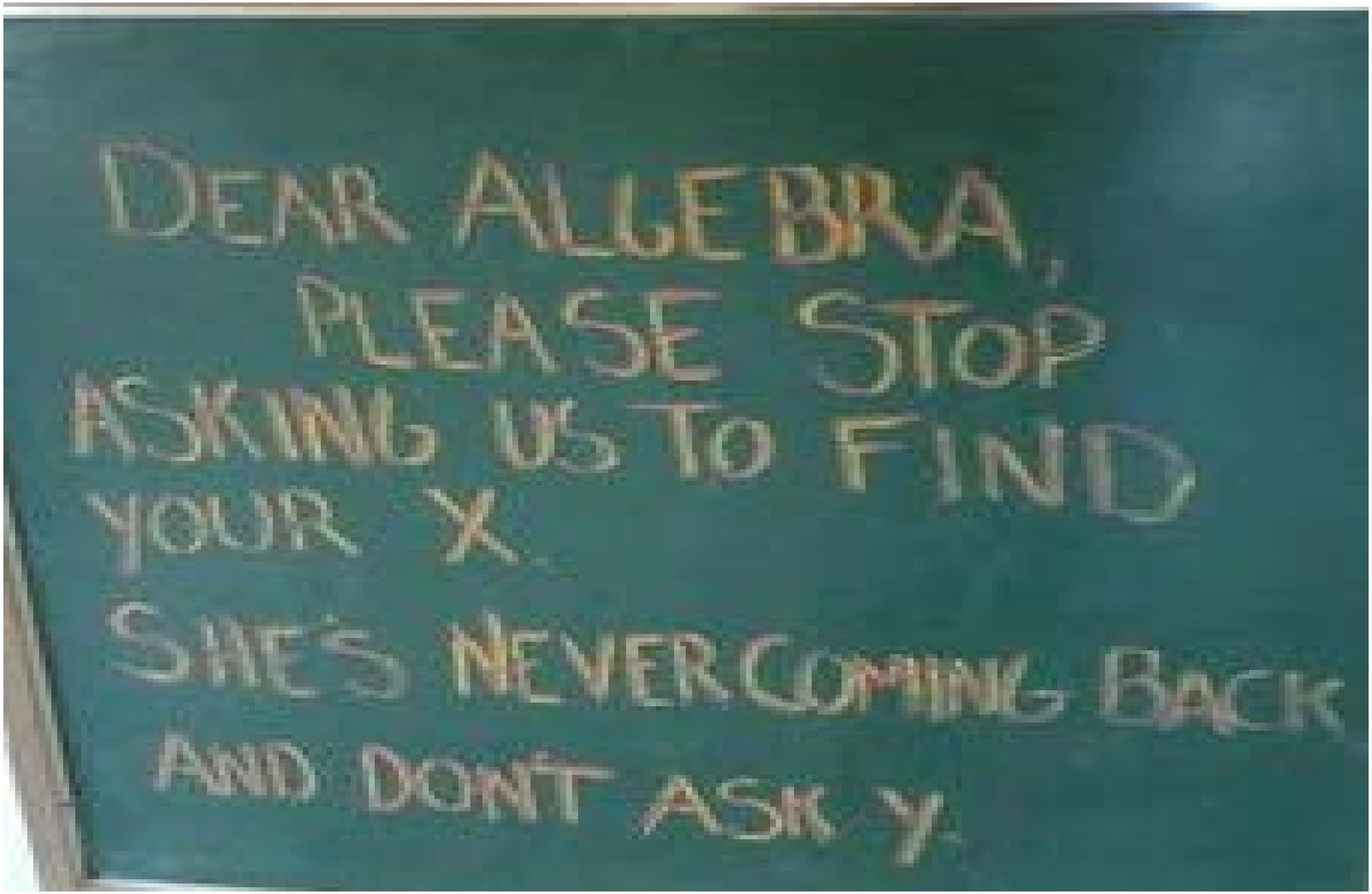
$x^2 = 16$

$\sqrt{x^2} = \sqrt{16}$

$x = 4$

Some giggles to end ….




DEAR ALGEBRA,
PLEASE STOP
ASKING US TO FIND
YOUR X
SHE'S NEVER COMING BACK
AND DON'T ASK Y.

Teacher Arrested at JFK

A public school teacher was arrested today at John F. Kennedy International Airport this morning as he attempted to board a flight while in possession of a ruler, a protractor, a compass, a slide-rule and a calculator. At a press conference just before noon today, Attorney General Eric Holder said he believes the man is a member of the notorious Al-Gebra movement. Although he did not identify the man, he confirmed the man has been charged by the FBI with carrying weapons of math instruction.

'Al-Gebra is a problem for us', the Attorney General said. 'They derive solutions by means and extremes, and sometimes go off on tangents in search of absolute values.' They use secret code names like "X" and "Y" and refer to themselves as "unknowns" but we have determined that they belong to a common denominator of the axis of medieval with coordinates in every country. As the Greek philosopher Isosceles used to say, "There are 3 sides to every triangle." The Attorney General went on to say "Teaching our children sentient thought processes and equipping them to solve problems is dangerous and puts our government at risk."

Solving Equations

Reflect on the learning intentions ….

- Transposing formulae
- Types of equations
- Solving linear equations