Mathematics for Medicine

Contents

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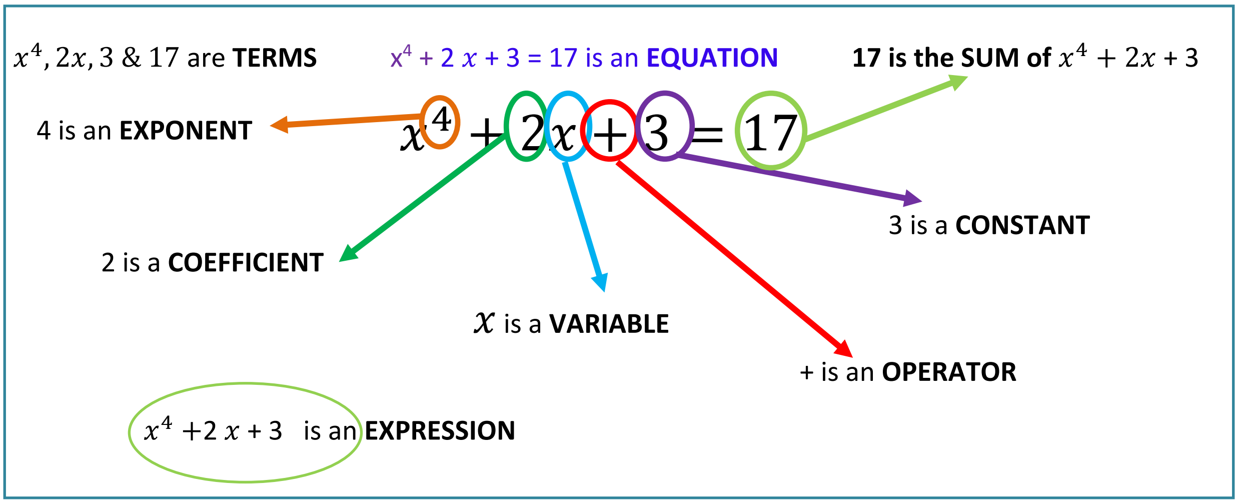
1. Terms and Operations

Glossary



Equation:	A mathematical sentence containing an equal sign. The equal sign demands that the expressions on either side are balanced and equal.
Expression:	An algebraic expression involves numbers, operation signs, brackets/parenthesis and variables that substitute numbers but does not include an equal sign.
Operator:	The operation $(+, -, \times, \div)$ which separates the terms.
Term:	Parts of an expression separated by operators which could be a number, variable or product of numbers and variables. E.g., $2x$, 3 & 17
Variable:	A letter which represents an unknown number. Most common is x, but can be any symbol.
Constant:	Terms that contain only numbers that always have the same value.
Coefficient:	A number that is partnered with a variable. The term $2x$ is a coefficient with variable. Between the coefficient and variable is a multiplication. Coefficients of 1 are not shown.
Exponent:	A value or base that is multiplied by itself a certain number of times. i.e., x^4 represents $x \times x \times x \times x$ or the base value x multiplied by itself 4 times. Exponents are also known as Powers or Indices.

In summary:

Variable:	x	Operator:	$+$
Constant:	3	Terms:	$3, 2x$ (a term with 2 factors) & 17
Equation:	$2x + 3 = 17$	Left hand expression:	$2x + 3$
Coefficient:	2	Right hand expression	17 (which is the sum of the LHE)

The symbols we use between the numbers to indicate a task or relationships are the **operators**, and the table below provides a list of common operators. You may recall the phrase, 'doing an operation.'

Symbol	Meaning
$+$	Add, Plus, Addition, Sum
$-$	Minus, take away, Subtract, Difference
$\times$	Times, Multiply, Product,
$\div$	Divide, Quotient
$\pm$	Plus and Minus
$\|a\|$	Absolute Value (ignore negative sign)
$=$	Equal
$\neq$	Not Equal
$<$	Less than
$>$	Greater than
$\ll$	Much Less than
$\gg$	Much More than
$\approx$	Approximately equal to
$\leq$	Less than or equal
$\geq$	Greater than or equal
Δ	Delta
$\sum$	Sigma (Summation)

Order of Operations

The **Order of Operations** is remembered using the mnemonic known as the BIDMAS or BOMDAS (Brackets, Indices or Other, Multiplication/Division, and Addition/Subtraction).

Brackets	{[( )]}
Indices or **O**ther	$x^2, \sin x, \ln x, etc$
Multiplication or **D**ivision	$\times$ or $\div$
Addition or **S**ubtraction	+ or -

The Rules:

1. Follow the order (BIMDAS, BOMDAS or BODMAS)
2. If two operations are of the same level, you work from left to right. E.g., $(\times \ or \ \div) \ or \ (+ \ or \ -)$
3. If there are multiple brackets, work from the inside set of brackets outwards. {[(   )]}

Example Problems:

1. Solve: $5 + 7 \times 2 + 5^2 =$

Step 1: 5^2 has the highest priority so: $5 + 7 \times 2 + 25 =$

Step 2: 7×2 has the next priority so: $5 + 14 + 25 =$

Step 3: only addition left, thus left to right: $19 + 25 = 44$

$\therefore 5 + 7 \times 2 + 5^2 = 44$

Question 1:

Here are some revision examples for practise:

a. $10 - 2 \times 5 + 1 =$

b. $10 \times 5 \div 2 - 3 =$

c. $12 \times 2 - 2 \times 7 =$

d. $48 \div 6 \times 2 - 4 =$

e. $What\ is\ the\ missing\ operation\ symbol\ 18 \_ 3 \times 2 + 2 = 14$

Check Answers p.25

2. Fractions – addition, subtraction, multiplication and division

Adding and subtracting fractions draws on the concept of equivalent fractions. The golden rule is that you can only **add and subtract fractions** if they have the **same denominator**, for example, $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$.

If two fractions do ***not*** have the same denominator, we must use equivalent fractions to find a "common denominator" before they can be added together.

In the example $\frac{1}{4}+\frac{1}{2}$, 4 is the lowest common denominator. Use the equivalent fractions concept to change $\frac{1}{2}$ $into$ $\frac{2}{4}$ by multiplying both the numerator and denominator by two: $\frac{1}{2}$ x $\frac{2}{2}$ = $\frac{2}{4}$

Now that the denominators are the same, addition or subtraction operations can be carried out.

In this example the lowest common denominator is found by multiplying 3 and 5, and then the numerators are multiplied by 5 and 3 respectively:

$$\frac{1}{3}+\frac{2}{5}=\frac{(1\times 5)+(2\times 3)}{(3\times 5)}=\frac{5+6}{15}=\frac{11}{15}$$

Compared to addition and subtraction, *multiplication and division of fractions* is easy to do, but sometimes a challenge to understand how and why the procedure works mathematically. For example, imagine I have $\frac{1}{2}$ of a pie and I want to share it between 2 people. Each person gets a quarter of the pie.

Mathematically, this example would be written as: $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$.

Remember that fractions and division are related; in this way, multiplying by a half is the same as dividing by two.

So $\frac{1}{2}$ (two people to share) of $\frac{1}{2}$ (the amount of pie) is $\frac{1}{4}$ (the amount each person will get).

But what if the question was more challenging: $\frac{2}{3}\times\frac{7}{16}$ =? This problem is not as easy as splitting pies.

A mathematical strategy to use is: "Multiply the numerators then multiply the denominators"

Therefore, $\frac{2}{3}\times\frac{7}{16}=\frac{(2\times 7)}{(3\times 16)}=\frac{14}{48}=\frac{7}{24}$

An alternative method you may recall from school is to simplify each term first. Remember, 'What we do to one side, we must do to the other.'

The first thing we do is look to see if there are any common multiples. For $\frac{2}{3}\times\frac{7}{16}=?$ we can see that 2 is a multiple of 16, which means that we can divide top and bottom by 2:

$$\frac{2^{\div 2}}{3}\times\frac{7}{16_{\div 2}}=\frac{1}{3}\times\frac{7}{8}=\frac{1\times 7}{3\times 8}=\frac{7}{24}$$

Division of fractions **seems odd, but it is a simple concept:**

You may recall the expression 'invert and multiply', which means we flip the divisor fraction (second term fraction). Hence, $\div \frac{1}{2}$ *is the same as* $\times \frac{2}{1}$

This 'flipped' fraction is referred to as the **reciprocal of the original fraction**.

Therefore, $\frac{2}{3} \div \frac{1}{2}$ is the same as $\frac{2}{3} \times \frac{2}{1} = \frac{(2\times2)}{(3\times1)} = \frac{4}{3} = 1\frac{1}{3}$ Note: dividing by half doubled the answer.

Question 2

1. Find the reciprocal of $2\frac{2}{5}$
2. $\frac{2}{3} \times \frac{7}{13} =$
3. $1\frac{1}{6} \times \frac{2}{9} =$
4. $\frac{3}{7} \div \frac{2}{5} =$
5. $2\frac{2}{5} \div 3\frac{8}{9} =$
6. $\frac{(-25)\div(-5)}{4-2\times7} =$
7. $\frac{-7}{2} \div \frac{-4}{9} =$
8. If we multiply 8 and the reciprocal or 2, what do we get?
9. Which is the better score in a physiology test; 17 out of 20 or 22 out of 25?
10. What fraction of H_2O_2 is hydrogen?
11. A patient uses a glass that holds $\frac{1}{5}$ of a jug's volume. The patient drinks eight full glasses during the day. What fraction of the second jug is left at the end of the day?

Check Answers p.25

3. Converting Decimals & Fractions

Converting Decimals into Fractions

Decimals are an almost universal method of displaying data, particularly given that it is easier to enter decimals, rather than fractions, into computers. But fractions can be more accurate. For example, $\frac{1}{3}$ is not 0.33 it is $0.3\dot{3}$

The method used to convert decimals into fractions is based on the notion of place value. The place value of the last digit in the decimal determines the denominator: tenths, hundredths, thousandths, and so on…

Example problems:

a) 0.5 has 5 in the tenth's column. Therefore, 0.5 is $\frac{5}{10} = \frac{1}{2}$ (simplified to an equivalent fraction).
b) 0.375 has the 5 in the thousandth column. Therefore, 0.375 is $\frac{375}{1000} = \frac{3}{8}$
c) 1.25 has 5 in the hundredths column and you have $1\frac{25}{100} = 1\frac{1}{4}$

The hardest part is converting to the lowest equivalent fraction. If you have a scientific calculator, you can use the fraction button (—). Read your manual if unsure.

If we take $\frac{375}{1000}$ from example 2 above:

Enter 375 then — followed by 1000 press = and answer shows as $\frac{3}{8}$.

NOTE: The calculator does not work for rounded decimals; especially thirds. E.g., 0.333 $\approx \frac{1}{3}$

This table lists some commonly encountered fractions expressed in their decimal form:

Decimal	Fraction	Decimal	Fraction
0.125	$\frac{1}{8}$	**0.5**	$\frac{1}{2}$
0.25	$\frac{1}{4}$	**0.66667**	$\frac{2}{3}$
0.33333	$\frac{1}{3}$	**.75**	$\frac{3}{4}$
0.375	$\frac{3}{8}$	**0.2**	$\frac{1}{5}$

Question 3:

Convert to fractions (no Calculator first, then check).

a) 0.65 =   b) 2.666 =   c) 0.54 =   d) 3.14 =

e) What is 40 multiplied by 0.2 (use your knowledge of fractions to solve)

Check Answers p. 25

Converting Fractions into Decimals

Converting fractions into decimals is based on place value. Using the concept of equivalent fractions, we can easily convert $\frac{2}{5}$ into a decimal. First, we convert to a denominator that has a 10 base:

$\frac{2}{5}$ into tenths $\rightarrow \frac{2^{\times 2}}{5_{\times 2}} = \frac{4}{10}$ $\therefore$ we can say that two fifths are the same as four tenths: 0.4

Converting a fraction to decimal form is a simple procedure because we simply use the divide key on the calculator.
Note: If you have a mixed number, convert it to an improper fraction before dividing it on your calculator.

<u>Example problems:</u>

- $\frac{2}{3} = 2 \div 3 = 0.66666666666\ldots \approx 0.67$
- $\frac{3}{8} = 3 \div 8 = 0.375$
- $\frac{17}{3} = 17 \div 3 = 5.6666666\ldots \approx 5.67$
- $3\frac{5}{9} = (27+5) \div 9 = 3.555555556\ldots \approx 3.56$

Questions 4

Convert to decimals. Round your answer to three decimal places where appropriate.

a. $\frac{17}{23} =$

b. $\frac{5}{72} =$

c. $56\frac{2}{3} =$

d. $\frac{29}{5} =$

Check Answers p. 25

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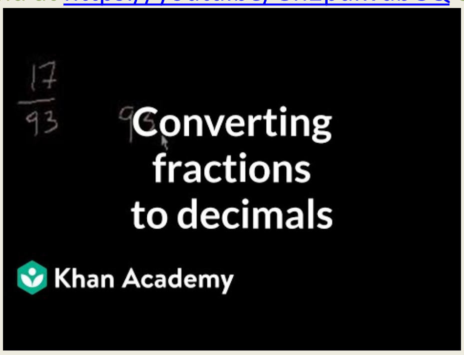
Watch this short Khan Academy video for further explanation of converting fractions to decimals. Found at https://youtu.be/Gn2pdkvdbGQ or if online just click below.



4. Percentage

The concept of percentage is an extension of the material we have already covered about fractions. To allow comparisons between fractions we need to use the same denominator. As such, all percentages use 100 as the denominator. The word percent or "per cent" means per 100. Therefore, 27% is $\frac{27}{100}$.

To use percentage in a calculation, the simple mathematical procedure is modelled below:

For example, 25% of 40 is $\frac{25}{100} \times 40 = 10$

Percentages are most used to compare parts of an original. For instance, the phrase '30% off sale,' indicates that whatever the original price, the new price is 30% less. However, the question might be more complex, such as, "How much is left?" or "How much was the original?"

Example problems:

a. An advertisement at the chicken shop states that on Tuesday everything is 22% off. If chicken breasts are normally $9.99 per kilo. What is the new per kilo price?

Step 1: SIMPLE PERCENTAGE:

$\frac{22}{100} \times 9.99 = 2.20$

Step 2: DIFFERENCE: Since the price is cheaper by 22%, $2.20 is subtracted from the original: $9.99 - 2.20 = \$7.79$

b. A new dress is now $237 reduced from $410. What is the percentage difference? As you can see, the problem is in reverse, so we approach it in reverse.

Step 1: DIFFERENCE: Since it is a discount the difference between the two is the discount. Thus, we need to subtract $237.00 from $410 to see what the discount was that we received.

$\$410 - \$237 = \$173$

Step 2: SIMPLE PERCENTAGE: now we need to calculate what percentage of $410 was $173, and so we can use this equation: $\frac{x}{100} \times 410 = 173$

We can rearrange the problem in steps: $\frac{x}{100} \times 410^{\div 410} = 173^{\div 410}$ this step involved dividing 410 from both sides to get $\frac{x}{100} = \frac{173}{410}$ Next we work to get the x on its own, so we multiply both sides by 100.

Now we have $x = \frac{173}{410} \times \frac{100}{1}$ Next, we solve, so 0.42 multiplied by 100, $\therefore$ 0.42×100 and we get 42.

$\therefore$ The percentage difference was 42%.

Let's check: 42% of $410 is $173, $410 - $173 = $237, the cost of the dress was $237.00 ✓.

Question 5

a) GST adds 10% to the price of most things. How much does a can of soft drink cost if it is 80c before GST?

b) When John is exercising his heart rate rises to 180 bpm. His resting heart rate is 70 % of this. What is his resting heart rate?

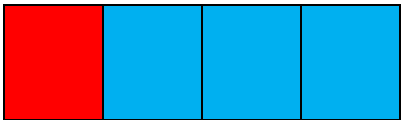
c) Which of the following is the largest? $\frac{3}{5}$ *or* $\frac{16}{25}$ *or* 0.065 *or* 63%? (Convert to percentages

Check Answers p. 25

5. Ratios

A ratio is a comparison of the size of one number to the size of another number. A ratio represents for every determined amount of one thing, how much there is of another thing. Ratios are useful because they are unit-less. That is, the relationship between two numbers remains the same regardless of the units in which they are measured.

Ratios use the symbol: to separate quantities being compared. For example, 1:3 means 1 unit to 3 units.



There is 1 red square to 3 blue squares

1:3

1 to 3

Ratios can be expressed as fractions, but you can see from the above diagram that 1:3 is not the same as $\frac{1}{3}$. The fraction equivalent is $\frac{1}{4}$

Example:

A pancake recipe requires flour and milk to be mixed to a ratio of 1:3. This means one part flour to 3 parts milk. No matter what device is used to measure, the ratio must stay the same.

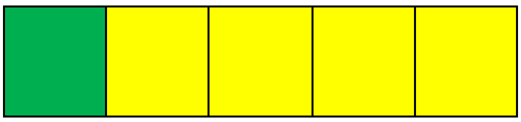
So, if I add 200 mL of flour, I add 200 mL x 3 = 600 mL of milk

If I add 1 cup of flour, I add 3 cups of milk

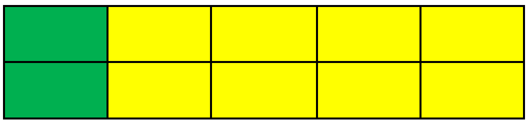
If I add 50 grams of flour, I add 150 grams of milk

Scaling ratios

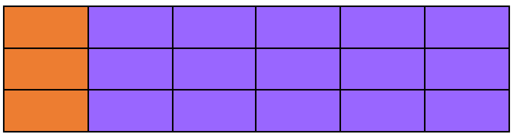
A ratio can be scaled up:



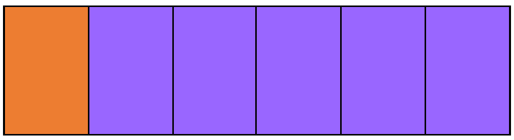
1:4 = 2:8



Or scaled down:



3:15 = 1:5

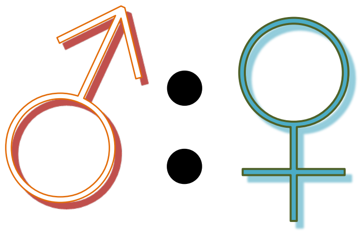


1:5 is the same as 2:10 is the same as 3:15 is the same as 4:20 and so on

Scaling ratios is useful in the same way that simplifying fractions can be helpful, for example, in comparing values. For ratios the same process as, simplifying fractions is applied – that is, scaling must be applied to both numbers.

For example, a first-year physiology subject has 36 males and 48 females, whereas the endocrinology subject has 64 males and 80 females. You are asked to work out which cohort has the largest male to female ratio.

The male: female ratios can be expressed as:



36:48 – physiology subject

64:80 – endocrinology subject

Both numbers of the ratio 36:48 can be divided by 12 to leave the ratio 3:4
Both numbers of the ratio 64:80 can be divided by 16 to leave the ratio 4:5

These two ratios cannot be easily directly compared, but they can be rescaled to a common value of 20 for the females, against which the males can be compared if they are rescaled equivalently.

3 (x5):4 (x5) = 15:20 – physiology subject

4 (x4):5 (x4) = 16:20 – endocrinology subject

Comparing the ratios now shows that the endocrinology subject has a slightly higher ratio of males to females.

Questions 6:

a) For a 1:5 concentration of cordial drink, how much cordial concentrate do I have to add to water to make up a 600 mL jug?

b) Jane reads 25 pages in 30 minutes. How long does it take her to read 200 pages?

c) A pulse is measured as 17 beats over 15 seconds. What is the heart rate per minute?

d) Which of the following ratios is the odd one out? 9:27. 3:9, 8:28, 25:75

Check Answers p. 25

Ratios and Percentages

Recall from the figures above, that the numbers in the ratio represents parts of a whole. To convert a ratio to percentage values, simply add the two parts of the ratio, to give the whole (total) and for each part, divide by the total. Then use the normal procedure to calculate the percentage by multiplying by 100.

Using the example from above, the table below calculates the percentage values of males and females for each subject from the ratios. Percentages allow for a quantified comparison.

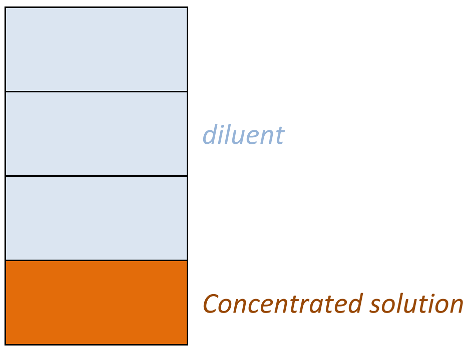
	Physiology 3:4	Endocrinology 4:5
Total	7	9
Male	$\frac{3}{7} \times 100 = 42.9\%$	$\frac{4}{9} \times 100 = 44.4\%$
Female	$\frac{4}{7} \times 100 = 57.1\%$	$\frac{5}{9} \times 100 = 55.6\%$

We are correct; there are more males than females (in percent) in the endocrinology subject.

Drug calculations using the expression <u>solute in diluent</u>

Sometimes drug ratios are written in the form of *solute **in** diluent*; for example, **1 <u>in</u> 4.**

This means 1 part of every 4 of the final volume is concentrated solution and this is mixed with 3 parts of the diluent. When expressed in this way the total parts are 4.



Question 7:

a) Write 2:3 in the form "2 in?"

b) Write a solution of 1 in 8 in the form of a ratio

c) How much concentrate do you need to make the following dilutions?

   a) 500 mL of a 1 in 4 solution   b) 600 mL of a 1:5 solution

d) Heparin, an anticoagulant, is available in a strength of 5000 units/mL. If 3000 units is required, what volume of heparin will be injected?

Check Answers p. 25

6. Algebra Refresh

Addition and Multiplication Properties

Maths Property	Rule	Example
Commutative **The number order for addition or multiplication doesn't affect the sum or product**	$a + b = b + a$ $ab = ba$	$1 + 3 = 3 + 1$ $2 \times 4 = 4 \times 2$
Associative **Since the Number order doesn't matter, it may be possible to regroup numbers to simplify the calculation**	$a + (b + c) = (a + b) + c$ $a(bc) = (ab)c$	$1 + (2 + 3) = (1 + 2) + 3$ $2 \times (2 \times 3) = (2 \times 2) \times 3$
Distributive **A factor outside the bracket can be multiplied with individual terms within a bracket to give the same result**	$a(b + c) = ab + ac$	$2(3 + 1) = 2 \times 3 + 2 \times 1$
Zero Factor	$a \times 0 = 0$ If $ab = 0$, then either $a = 0$ or $b = 0$	$2 \times 0 = 0$
Rules for Negatives	$-(-a) = a$ $(-a)(-b) = ab$ $-ab = (-a)b = a(-b)$ $= -(ab)$ $(-1)a = -a$	$-(-3) = 3$ $(-2)(-3) = 2 \times 3$ $-2 \times 3 = (-2) \times 3$ $= 2 \times (-3)$ $= -(2 \times 3)$ $(-1) \times 2 = -2$
Rules for Division	$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ $\frac{-a}{-b} = \frac{a}{b}$ $If\ \frac{a}{b} = \frac{c}{d}\ then\ ad = bc$ $Proof:$ $\frac{a}{b} = \frac{c}{d}$ $\frac{\not{b} \times a}{\not{b}} = \frac{bc}{d}$ (multiply everything by b) $a = \frac{bc}{d}$ $a \times d = \frac{bc \times \not{d}}{\not{d}}$ (multiply by d) $ad = bc$	$-\frac{4}{2} = \frac{-4}{2} = \frac{4}{-2}$ $\frac{-6}{-3} = \frac{6}{3}$ $If\ \frac{1}{2} = \frac{3}{4}\ then\ 1 \times 4$ $= 2 \times 3$

Question 8:

a) Simplify the following

i. $3(x-1) = 2(x+5)$

ii. $2x + 8 = \frac{1}{2}(43 + x)$

iii. $2x^2 - 3x^3 - x^2 + 2x =$

iv. $\frac{2(4+6)}{2+3} =$

b) Expand $2y^2(3x + 7y + 2)$

c) What is the molecular mass of H_2O_2 Given *O = 16 and H = 1?*

d) Calculate the molecular mass of $CuSO_4$ $7H_2O$ hydrated copper sulphate. Given *Cu = 64, S = 32, O = 16 and H = 1*

Check Answers p. 26

7. Power Operations

Powers are also called **exponents** or **indices**; we can work with the **indices** to simplify expressions and to solve problems.

Some key ideas:

a) Any base number raised to the power of 1 is the base itself: for example, $5^1 = 5$
b) Any base number raised to the power of 0 equals 1, so: $4^0 = 1$
c) Powers can be simplified if they are **multiplied** or **divided** and have the **same** base.
d) Powers of powers are multiplied. Hence, $(2^3)^2 = 2^3 \times 2^3 = 2^6$
e) A negative power indicates a reciprocal: $3^{-2} = \frac{1}{3^2}$

Certain rules apply and are often referred to as: **Index Laws.**

Below is a summary of the index rules:

Index Law	Substitute variables for values
$\boldsymbol{a^m \times a^n = a^{m+n}}$	$2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$
$\boldsymbol{a^m \div a^n = a^{m-n}}$	$3^6 \div 3^3 = 3^{6-3} = 3^3 = 27$
$\boldsymbol{(a^m)^n = a^{mn}}$	$(4^2)^5 = 4^{2\times5} = 4^{10} = 1048\,576$
$\boldsymbol{(ab)^m = a^m b^m}$	$(2 \times 5)^2 = 2^2 \times 5^2 = 4 \times 25 = 100$
$\boldsymbol{({}^{a}/_{b})^m = a^m \div b^m}$	$(10 \div 5)^3 = 2^3 = 8;\ (10^3 \div 5^3) = 1000 \div 125 = 8$
$\boldsymbol{a^{-m} = \frac{1}{a^m}}$	$4^{-2} = \frac{1}{4^2} = \frac{1}{16}$
$\boldsymbol{a^{\frac{1}{m}} = \sqrt[m]{a}}$	$8^{1/3} = \sqrt[3]{8} = 2$
$\boldsymbol{a^0 = 1}$	$6^3 \div 6^3 = 6^{3-3} = 6^0 = 1; (6 \div 6 = 1)$

EXAMPLE PROBLEMS:

a) Simplify $6^5 \times 6^3 \div 6^2 \times 7^2 + 6^4 =$

$= 6^{5+3-2} \times 7^2 + 6^4$

$= 6^6 \times 7^2 + 6^4$

b) Simplify $g^5 \times h^4 \times g^{-1} =$

$= g^5 \times g^{-1} \times h^4$

$= g^4 \times h^4$

Watch this short Khan Academy video for further explanation: **“Multiplying and Dividing Powers” click below or** https://youtu.be/CZ5ne_mX5_I

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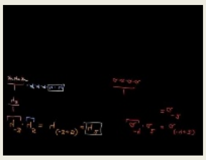


Question 9:

a) *Apply the index laws/rules:*

i. Simplify $5^2 \times 5^4 + 5^2 =$

ii. Simplify $x^2 \times x^5 =$

iii. Simplify $4^2 \times t^3 \div 4^2 =$

iv. Simplify $(5^4)^3 =$

v. Simplify $\frac{2^4 3^6}{3^4} =$

vi. Simplify $3^2 \times 3^{-5} =$

vii. Simplify $\frac{9(x^2)^3}{3xy^2} =$

viii. Simplify $a^{-1}\sqrt{a} =$

b) What is the value of x for the following?

i. $49 = 7^x$

ii. $\frac{1}{4} = 2^x$

iii. $88 = 11^1 \times 2^x$

iv. $480 = 2^x \times 3^1 \times 5^1$

v. Show that $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$

Check Answers p. 26

8. Scientific Notation

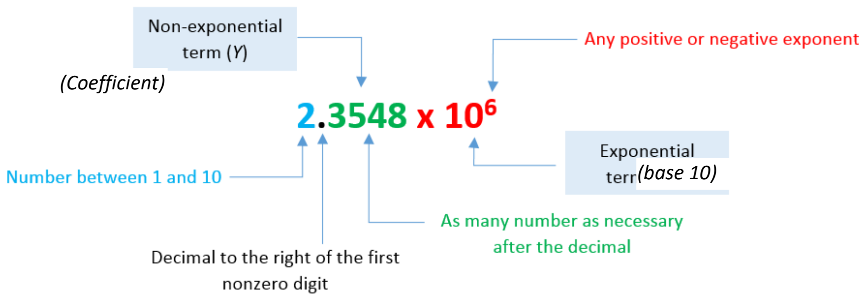
Numbers as multiples or fractions of ten	Number	Number as a power of ten
10 x 10 x 10	1000	10^3
10 x 10	100	10^2
10	10	10^1
10 x 1/10	1	10^0
1/10	0.1	10^{-1}
1/100	0.01	10^{-2}
1/1000	0.001	10^{-3}

Scientific notation is a convenient method of representing and working with very large and very small numbers. Transcribing a number such as 0.000000000000082 or 5480000000000 can be frustrating since there will be a constant need to count the number of zeroes each time the number is used. Scientific notation provides a way of writing such numbers easily and accurately.

Scientific notation requires that a number is presented as a non-zero digit followed by a decimal point and then a power (exponential) of base 10. The exponential is determined by counting the number places the decimal point is moved.

The number 65400000000 in scientific notation becomes 6.54×10^{10}.

The number 0.00000086 in scientific notation becomes 8.6×10^{-7}. (Note: $10^{-6} = \frac{1}{10^6}$.)



If *n* is positive, shift the decimal point that many places to the right.

If *n* is negative, shift the decimal point that many places to the left.

Question 10:

Write the following in scientific notation:

a. 450

b. 90000000

c. 3.5

d. 0.0975

Write the following numbers out in full:

e. 3.75×10^2

f. 3.97×10^1

g. 1.875×10^{-1}

h. -8.75×10^{-3}

Check Answers p. 26

9. Calculations with Scientific Notation

Multiplication and *division* calculations of quantities expressed in scientific notation follow the index laws since they all they all have the common base, i.e., base 10.

Here are the steps:

Multiplication	Division
A. **Multiply** the coefficients	1. **Divide** the coefficients
B. **Add** their exponents	2. **Subtract** their exponents
C. Convert the answer to scientific Notation	3. Convert the answer to scientific Notation
Example: $(7.1 \times 10^{-4}) \times (8.5 \times 10^{-5})$ $7.1 \times 8.5 = 60.35$ (multiply coefficients) $10^{-4} \times 10^{-5} = 10^{(-4+-5)=-9}$ (add exponents) $= 60.35 \times 10^{-9}$ – check it's in scientific notation ✗ $= 6.035 \times 10^{-8}$ – convert to scientific notation ✓	**Example:** $(9 \times 10^{20}) \div (3 \times 10^{11})$ $9 \div 3 = 3$ (divide coefficients) $10^{20} \div 10^{11} = 10^{(20-11)=9}$ (subtract exponents) $= 3 \times 10^{9}$ – check it's in scientific notation ✓

Recall that addition and subtraction of numbers with exponents (or indices) requires that the base and the exponent are the same. Since all numbers in scientific notation have the same base 10, for *addition* and *subtraction* calculations, we must adjust the terms, so the exponents are the same for both. This will ensure that the digits in the coefficients have the correct place value so they can be simply added or subtracted. Here are the steps:

Addition	Subtraction
1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent	1. Determine how much the smaller exponent must be increased by so it is equal to the larger exponent
2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places	2. Increase the smaller exponent by this number and move the decimal point of the coefficient to the left the same number of places
3. Add the new coefficients	3. Subtract the new coefficients
4. Convert the answer to scientific notation	4. Convert the answer to scientific notation
Example: $(3 \times 10^{2}) + (2 \times 10^{4})$ $4 - 2 = 2$ increase the small exponent by 2 to equal the larger exponent 4 0.03×10^{4} the coefficient of the first term is adjusted so its exponent matches that of the second term $= (0.03 \times 10^{4}) + (2 \times 10^{4})$ the two terms now have the same base and exponent and the coefficients can be added $= 2.03 \times 10^{4}$ check it's in scientific notation ✓	**Example:** $(5.3 \times 10^{12}) - (4.224 \times 10^{15})$ $15 - 12 = 3$ increase the small exponent by 3 to equal the larger exponent 15 0.0053×10^{15} the coefficient of the first term is adjusted so its exponent matches that of the second term $= (0.0053 \times 10^{15}) - (4.224 \times 10^{15})$ the two terms now have the same base and exponent and the coefficients can be subtracted. $= -4.2187 \times 10^{15}$ check it's in scientific notation ✓

Questions 11:

1. $(4.5 \times 10^{-3}) \div (3 \times 10^{2})$

2. $(2.25 \times 10^{6}) \times (1.5 \times 10^{3})$

3. $(6.078 \times 10^{11}) - (8.220 \times 10^{14})$ (give answer to 4 significant figures).

4. $(3.67 \times 10^{5}) \times (23.6 \times 10^{4})$

5. $(7.6 \times 10^{-3}) + (\sqrt{9.0 \times 10^{-2}})$

6. Two particles weigh 2.43 X 10^{-2} grams and 3.04 X 10^{-3} grams. What is the difference in their weight in scientific notation?

7. How long does it take light to travel to the Earth from the Sun in seconds, given that the Earth is 1.5 X 10^{8} km from the Sun and the speed of light is 3 X 10^{5} km/s?

Check Answers p. 26

10. Units and unit Conversion

Measurement is used every day to describe quantity. There are various types of measurements such as time, distance, speed, weight and so on. There are also various systems of units of measure, for example, the Metric system and the Imperial system. Within each system, for each base unit, other units are defined to reflect divisions or multiples of the base unit. This is helpful for us to have a range of unit terms that reflect different scale

Measurements consist of two parts – the number and the identifying unit.

Number → **50 kg** ← Identifying unit

In scientific measurements, units derived from the metric system are the preferred units. The metric system is a decimal system in which larger and smaller units are related by factors of 10.

Table 1: Common Prefixes of the Metric System

Prefix	**Abbreviation**	**Relationship to Unit**	**Exponential Relationship to Unit**	**Example**
mega-	M	1 000 000 x Unit	10^{6} x Unit	2.4ML -Olympic sized swimming pool
kilo-	k	1000 x Unit	10^{3} x Unit	The average newborn baby weighs 3.5kg
-	-	Units	Unit	meter, gram, litre, sec
deci-	d	1/10 x Unit or 0.1 x Unit	10^{-1} x Unit	2dm - roughly the length of a pencil
centi-	c	1/100 x Unit or 0.01 x Unit	10^{-2} x Unit	A fingernail is about 1cm wide
milli-	m	1/1000 x Unit or 0.001 x Unit	10^{-3} x Unit	A paperclip is about 1mm thick
micro-	µ	1/1 000 000 x Unit or 0.000001 x Unit	10^{-6} x Unit	human hair can be up to 181 µm
nano-	n	1/1 000 000 000 x Unit or 0.000000001 x Unit	10^{-9} x Unit	DNA is 5nm wide

Table 2: Common Metric Conversions

Unit	Larger Unit	Smaller Unit
1 metre	1 kilometre = 1000 meters	100 centimetres = 1 meter 1000 millimetres = 1 meter
1 gram	1 kilogram = 1000 grams	1000 milligrams = 1 gram 1 000 000 micrograms = 1 gram
1 litre	1 kilolitre = 1000 litres	1000 millilitres = 1 litre

Often, we are required to convert a unit of measure; we may be travelling and need to convert measurements from imperial to metric (e.g., mile to kilometres), or we may need to convert units of different sale (e.g., millimetres to metres) for ease of comparison. In the fields of science and medicine, converting measurement can be a daily activity.

It helps to apply a formula to convert measurement. However, it is essential to understand the how and why of the formula otherwise the activity becomes one we commit to memory without understanding what is really happening. Most mistakes are made when procedures are carried out without understanding the context, scale or purpose of the conversion.

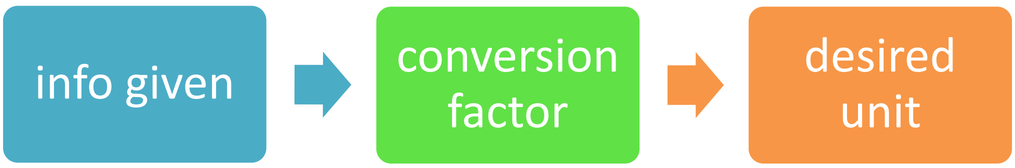
Unit Conversion rules:

I. *Always write the unit of measure associated with every number.*
II. *Always include the units of measure in the calculations.*

Unit conversion requires algebraic thinking (see Maths Refresher Workbook 2).

Let's convert 58mm into metres. $58\ mm \times \frac{1\ m}{1000\ mm} = 0.058\ m$

The quantity $\frac{1\ m}{1000\ mm}$ is called a **conversion factor;** it is a division/quotient; in this case it has metres on top and *mm* on the bottom. So, we can see that we work with information given, a conversion factor, and then the desired unit.



Example problem:

Convert 125 milligrams (mg) to grams (g). There are 1000 mg in a gram, so our conversion factor is $\frac{1\,g}{1000\,mg}$

The working is as follows: $125\,\cancel{mg} \times \frac{1\,g}{1000\,\cancel{mg}} = 0.125\,g$

Here we can cancel out the mg and are left with g which is the unit we were asked to convert to.

It is helpful to have a thinking process to follow. This one comes from the book, *Introductory Chemistry* (Trod, 2011, pp. 25-35). There are four steps to the process: sort, strategize, solve and check.

1. **Sort:** First we sort out what given information is available.
2. **Strategize**: The second step is where a conversion factor is created. You may need to look at a conversion table and possibly use a combination of conversion factor.
3. **Solve**: This is the third step which simply means to solve the problem.
4. **Check**: The last step requires some logical thought; does the answer make sense?
5.

Example problem: Convert 2 kilometres (km) into centimetres (cm).

- **Sort**: we know there are 1000 metres in one km, and 100cm in one metre.
- **Strategize**: So, our conversion factors could be $\frac{1000m}{1km}$ and $\frac{100cm}{1m}$
- **Solve**: $2\,km \times \frac{1000\,m}{1km} \times \frac{100\,cm}{1m} = x\,cm$

  $2\,\cancel{km} \times \frac{1000\,\cancel{m}}{1\cancel{km}} \times \frac{100\,cm}{1\cancel{m}} = 2 \times 1000 \times 100\,cm \;\therefore 2\,km = 200{,}000\,cm$
- **Check**: is there 200,000cm in a kilometre? Yes, that seems sensible.

Question 12:

Convert the following:

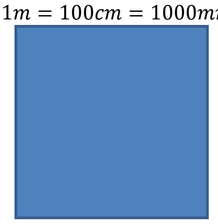
a) 285m into kilometres

b) 0.15g to milligrams

c) Using this information: $1\,m^2 = 10000cm^2 = 1000000mm^2$
Convert $1.5m^2$ $into$ mm^2

- Sort
- Strategize
- Solve

$1m = 100cm = 1000mm$



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More Examples:

a) Convert 0.15 g to kilograms and milligrams	b) Convert 5234 mL to litres
Because 1 kg = 1000 g, 0.15 g can be converted to kilograms as shown: $\mathbf{0.15\ g\ x\ \frac{1\ kg}{1000\ g} = 0.00015\ kg}$ **Also, because 1 g = 1000 mg, 0.15 g can be converted to milligrams as shown:** $\mathbf{0.15\ g\ x\ \frac{1000\ mg}{1\ g} = 150\ mg}$	Because 1 L = 1000 mL, 5234 mL can be converted to litres as shown: $\mathbf{5243\ mL\ x\ \frac{1\ L}{1000\ mL} = 5.234\ L}$

Question 13:

1. Convert 120 g to kilograms and milligrams. Use scientific notation for your answer.
2. Convert 4.264 L to kilolitres and millilitres
3. Convert 670 micrograms to grams. Give your answer in scientific notation.
4. How many millilitres are in a cubic metre?
5. How many inches in 38.10cm (2.54cm = 1 inch?
6. How many centimetres in 1.14 kilometres?
7. How many litres are in 3.5×10^5 millilitres?

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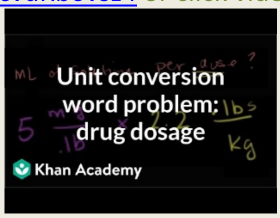
Watch this short Khan Academy video for further explanation: "Unit Conversion word problem" go to https://youtu.be/ScvuRb6vsz4 or click video below.



11. Logarithms

As a refresher…

4^2

i. The 2 is called the **exponent, index** or **power**.
ii. The 4 is called the **base**.

With roots we tried to find the unknown base. Such as, $x^3 = 64$ is the same as $\sqrt[3]{64} = x$; (x is the base).

A **logarithm** is used to find an unknown **power/exponent**. For example, $4^x = 64$ is the same as $\log_4 64 = x$
This example above is spoken as: 'The logarithm of 64 with base 4 is x.' The base is written in subscript.

The general rule is: $N = b^x \Leftrightarrow \log_b N = x$

i. In mathematics the base can be any number, but only two types are commonly used:
   a. $\log_{10} N$ $(base\ 10)$ is often abbreviated as simply Log, and
   b. $\log_e N$ $(base\ e)$ is often abbreviated as Ln or natural log
ii. $\log_{10} N$ is easy to understand for: $\log_{10} 1000 = \log 1000 = 3$ $(10^3 = 1000)$
   $\log 100 = 2$
iii. Numbers which are not 10, 100, 1000 and so on are not so easy. For instance, $\log 500 = 2.7$ It is more efficient to use the calculator for these types of expressions.

Refer to the Index Laws

Question 14:

Write the logarithmic form for:

a) $5^2 = 25$
b) $6^2 = 36$
c) $3^5 = 243$

Use your calculator to solve

d) Log 10000 =

e) Log 350 =

f) Ln 56 =

g) Ln 100 =

Check Answers p. 27

Some log rules for your reference

$\log_b(xy) = \log_b(x) + \log_b(y)$

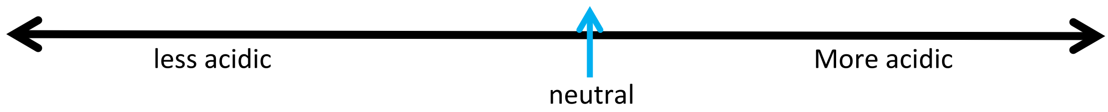
$\log_b(x/y) = \log_b(x) - \log_b(y)$

$\log_b(x^y) = y \cdot \log_b(x)$

Let's explore further

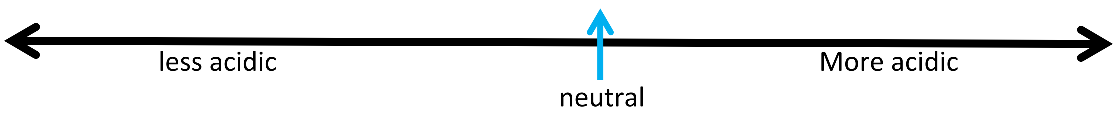
Logs to the base 10 are used in chemistry where very small concentrations are involved. For instance, acidity is determined by the concentration of H^+ ions (measured in moles/litre and written as $[H^+]$ in a solution. In pure water $[H^+]$ = 0.0000001 mole/litre of 1×10^{-7}

$[H^+]$ = 10^{-14} 10^{-13} 10^{-12} 10^{-11} 10^{-10} 10^{-9} 10^{-8} 10^{-7} 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1}



A special scale called pH has been developed to measure acidity and it is simply the 'negative index' of the above scale. You may have noticed this scale if you help with the balancing a swimming pool to ensure the water is safe for swimming.

pH = 14 13 12 11 10 9 8 7 6 5 4 3 2 1



So, a solution with a pH=7 is neutral, while pH =3 is acidic, pH=2 is more acidic, pH 12 is alkaline and so on.

Question 15

Use the scales above to answer these questions

Convert these $[H^+]$ concentrations to scientific notation and then pH.

a) $[H^+]$ = 0.001 moles/litre

b) $[H^+]$ = 0.00000001 moles/litre

c) $[H^+]$ = 0.0000001 moles/litre

d) $[H^+]$ = 0.000000000001 moles/litre

e) When converting from the $[H^+]$ scale to the pH scale, we take "the negative of the log to base 10 of the $[H^+]$." Can you translate this English statement into a mathematical equation?

f) $pH = -\log_{10}[H^+]$ if $[H^+] = 0.1$ what is the pH?

g) If the solution in part f above is diluted by a factor of 10 what will be the pH now?

h) *If the pH is 3.30 what is* $[H^+]$

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12. Answers

Q 1. Order of Operations

a) 1
b) 22
c) 10
d) 12
e) ×

Q2. Fraction Multiplication and Division

1. $\frac{5}{12}$
2. $\frac{14}{39}$
3. $\frac{7}{27}$
4. $1\frac{1}{14}$
5. $\frac{108}{175}$
6. $-\frac{1}{2}$
7. $\frac{63}{8}$ or $7\frac{7}{8}$
8. 4
9. 22 out of 25
10. $\frac{1}{2}$
11. $\frac{2}{5}$

Q3. Converting Decimals into Fractions.

a. $\frac{65}{100} = \frac{13}{20}$

b. $2\frac{666}{1000} \approx= 2\frac{2}{3}$

c. $\frac{54}{100} = \frac{27}{50}$

d. $3\frac{14}{100} = 3\frac{7}{50}$

e. $\frac{40}{5} = 8$

Q4. Converting Fractions into Decimals

a. 0.739 b. 0.069 c. 56.667 d. 5.8

Q5. Percentage

a. 88c b. 126 c. $\frac{16}{25}$

Q6. Ratios

a) 100 mL
b) 240 minutes = 4 hours
c) 68
d) 8:28

Q7. Ratios – solute in dilutant

a) 2 in 5 b) 1:7 c) 125 mL ii. 100 mL d) 0.6 mL

Q8. Algebra

a)

i. $x = 13$

ii. $x = 9$

iii. $x(x - 3x^2 + 2$

iv. 4

b) $6y^2x + 14y^3 + 4y^2$

c) 34

d) 286

Q9. Power Operations

a)

i. $5^2 \times 5^4 + 5^2 = 5^6 + 5^2$

ii. $x^2 \times x^5 = x^7$

iii. $4^2 \times t^3 \div 4^2 = t^3$

iv. $(5^4)^3 = 5^{12}$

v. $\frac{2^4 3^6}{3^4} = 2^4 3^2 = 16 \times 9 = 144$

vi. $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{27}$

vii. $\frac{9(x^2)^3}{3xy^2} = \frac{9x^6}{3xy^2} = \frac{3x^5}{y^2}$

viii. $a^{-1}\sqrt{a} = a^{-1} \times a^{\frac{1}{2}} = a^{-\frac{1}{2}} = \frac{1}{\sqrt{a}}$ or $\frac{1}{a^{\frac{1}{2}}}$

b)

i. $x = 2$

ii. $x = -2$

iii. $x = 3$

iv. $x = 5$

v. Show that $\frac{16a^2b^3}{3a^3b} \div \frac{8b^2a}{9a^3b^5} = 6ab^5$ $\frac{16a^2b^3}{3a^3b} \times \frac{9a^3b^5}{8b^2a}$

$= \frac{2a^5b^8 \times 3}{b^3a^4} = \frac{2a^1b^5 \times 3}{1} = 6ab^5$

Q10. Scientific Notation

a) 4.5×10^2

b) 9.0×10^7

c) 3.5×10^0

d) 9.75×10^{-2}

e) 375

f) 39.7

g) 0.1875

h) 0.00875

Q11. Calculations

a) 1.5×10^{-5}

b) 3.375×10^9

c) -8.214×10^{14}

d) 8.8612×10^{10}

e) 3.076×10^{-2}

f) 2.126×10^{-2}

g) 500 s

Q12. Unit Conversion

a) 0.285 km

b) 150 mg

c) $1.5 \times 10^6\ mm^2$

Q13 Unit Conversion

a) $1.2 \times 10^8\ mg$

b) $4.264 \times 10^{-3}\ kL$ & 4264 mL

c) $6.7 \times 10^{-4}\ g$

d) 1×10^6 mL

e) 15 inches

f) 114 000 cm

g) 3500 L

Q14. Logarithms

a) $\log_5 25 = 5$

b) $\log_6 36 = 2$

c) $\log_3 243 = 5$

Solve and write in exponential form:

d) 4

e) 2.54

f) 4.03

g) 4.61

Q15 Conversion to pH

a) $[H^+]$ = 0.001 moles/litre = 10^{-3} pH=3

b) $[H^+]$ = 0.00000001 moles/litre = 10^{-8} pH=8

c) $[H^+]$ = 0.0000001 moles/litre = 10^{-7} pH=7

d) $[H^+]$ = 0.000000000001 moles/litre = 10^{-12} pH12

e) pH= $-\log_{10} [H^+]$

f) 1

g) 2

h) 5 X 10^{-4}