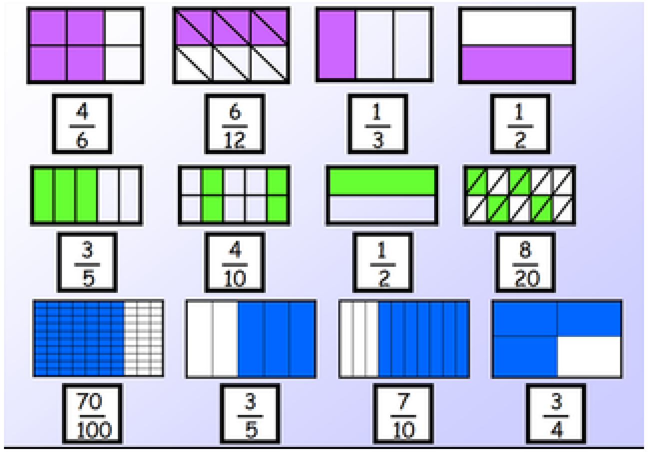
Maths Refresher

Working with Fractions

Working with fractions

Learning intentions ….

- Become familiar with fractions
- Equivalent fractions
- Converting mixed numbers to improper fractions
- Converting improper fractions to mixed numbers
- Converting decimals into fractions
- Converting fractions into decimals
- Fraction addition and subtraction
- Fraction multiplication and division



http://www.3to11maths.com/equivalent-fractions.html

Become familiar with fractions

- Fractions are representations of "parts of a whole."
- Fractions are '**rational**' numbers
  - A rational number can take the form of a fraction $\frac{a}{b}$
  - the numerator and the denominator are whole numbers
- An **'irrational'** number cannot be written as a simple fraction. For example:

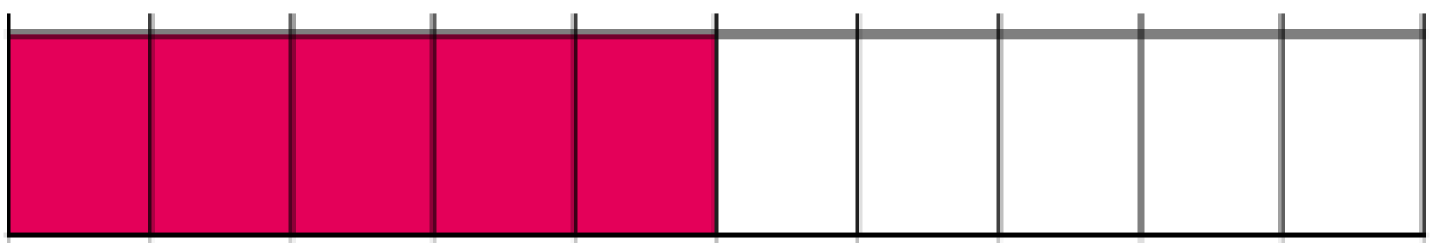
π = 3.1415926535897932384626433832795...

Become familiar with fractions

- It is important to recognise that division and fractions are linked.
- Even the division symbol ($\div$) is a fraction:

  $\frac{1}{2}$ is the same as 1 divided by 2

  $1 \div 2 = 0.5$



Become familiar with fractions

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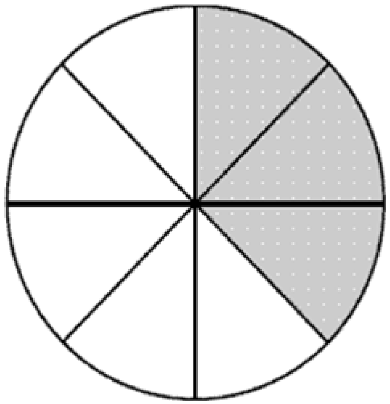
- A fraction is made up of two main parts: **numerator** and **denominator**
- $\frac{3}{4} \rightarrow \frac{numerator}{denominator}$ → the "bar" or vinculum
- The denominator represents how many even parts of the whole has been divided into.
- The numerator tells you the **number** of even parts you have.
- For example, $\frac{5}{8}$ of a pizza means:
  - **Denominator**: you have cut the pizza into 8 even pieces
  - **Numerator**: you have 5 of them.
  - Then there will be $\frac{3}{8}$ left over.



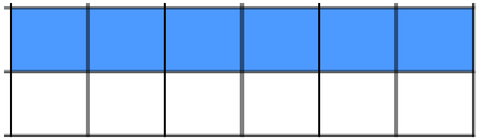
Become familiar with fractions

- A **proper** fraction has a numerator smaller than the denominator, e.g. $\frac{3}{4}$
- An **improper** fraction has a numerator larger than the denominator, e.g. $\frac{4}{3}$
- A **mixed** fraction is made up from a whole number and a fraction, e.g. $2\frac{1}{3}$

"Introduction to fractions"
https://www.khanacademy.org/math/arithmetic/fractions/understanding_fractions/v/introduction-to-fractions

Equivalent fractions

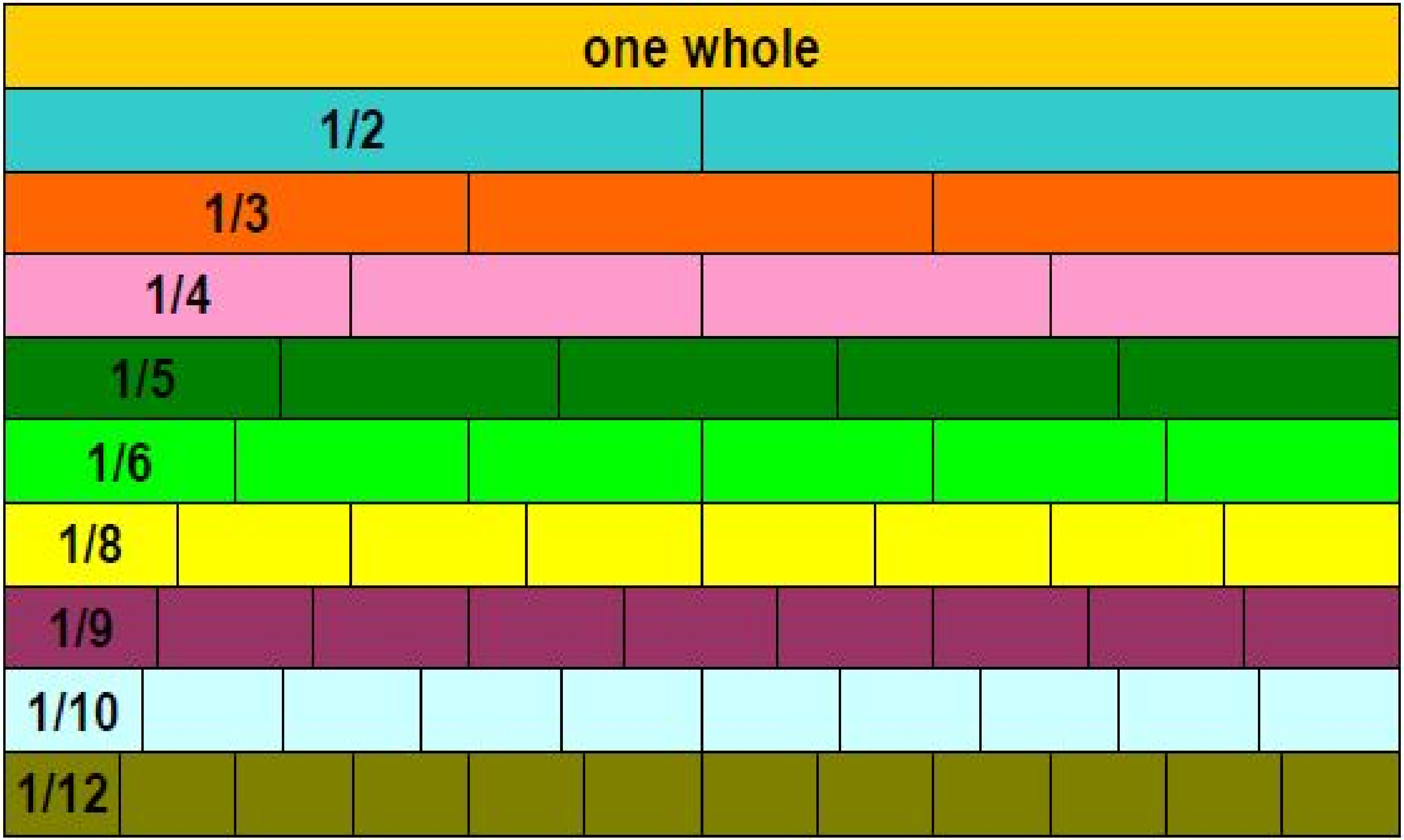
- Fractions should always be displayed in their simplest form,
  - $\frac{6}{12}$ is written as $\frac{1}{2}$



- Equivalence is a concept that is easy to understand when a fraction wall is used – on the next slide.
- You will see that each row has been split into different fractions:
  top row into 2 halves, bottom row 12 twelfths.
- An equivalent fraction splits the row at the same place. Therefore:

  » $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$

Equivalent fractions



Equivalent fractions

- Mathematically, whatever I do to the numerator (multiply or divide), I must also do to the denominator (and vice versa).

- Take $\frac{2}{3}$ as an example. If I multiply the numerator by 4, then I must multiply the denominator by 4.

$$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$$

Equivalent fractions

- To **simplify a fraction**, you must divide both the numerator and the denominator by the same number.
- For example:

$$\frac{8^{(\div 4)}}{12_{(\div 4)}} = \frac{2}{3}$$

"Equivalent fractions"
https://www.khanacademy.org/math/arithmetic/fractions/Equivalent_fractions/v/equivalent-fractions

Your turn ….

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EQUIVALENT FRACTIONS

Example problems:

1. $\frac{3}{5} = \frac{\square}{20}$ Answer: The denominator was multiplied by 4. (20 ÷5 =4)

   So, the numerator must also be multiplied by 4. $\therefore \frac{3}{5} = \frac{12}{20}$

2. $\frac{27}{81} = \frac{9}{\square}$ Answer: The numerator was divided by 3. (27 ÷9 =3)

   So, the denominator must also be divided by 3. $\therefore \frac{27}{81} = \frac{9}{27}$

Practise problems:

1. $\frac{2}{3} = \frac{\square}{9}$
2. $\frac{5}{7} = \frac{45}{\square}$
3. $\frac{9}{10} = \frac{\square}{30}$
4. $\frac{\square}{52} = \frac{4}{13}$

Equivalent fractions

Answers:

1. $\frac{2}{3} = \frac{6}{9}$

2. $\frac{5}{7} = \frac{45}{63}$

3. $\frac{9}{10} = \frac{27}{30}$

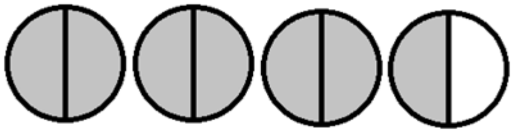
4. $\frac{16}{52} = \frac{4}{13}$

Converting mixed numbers to improper fractions

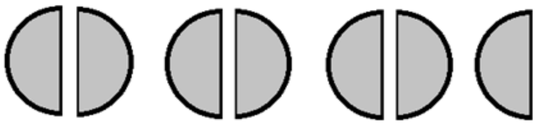
- A **mixed number** is a way of expressing quantities greater than 1.
- A mixed number represents the number of wholes and **remaining** parts of a whole that you have, while an improper fraction represents how many **parts** you have.
- The diagram on the next slide illustrates the difference between a mixed number and an improper fraction, using a quantity of car oil as an example.

"Mixed numbers and improper fractions" (converting both ways)
https://www.khanacademy.org/math/pre-algebra/fractions-pre-alg/mixed-numbers-pre-alg/v/mixed-numbers-and-improper-fractions
Mr Duey "Improper fractions"
https://www.youtube.com/watch?v=V96_PjlrVQc

Converting mixed numbers to improper fractions



is the same as



$$3\frac{1}{2} = 7\ halves = \frac{7}{2}$$

On the left, we use a mixed number to represent 3 whole litres and 1 half litre. We write this mixed number as 3 ½. On the right, we use an improper fraction to represent 7 half litres. We write this improper fraction as $\frac{7}{2}$.

Converting mixed numbers to improper fractions

- You are more likely to encounter mixed numbers than improper fractions in every day language. EG: you are more likely to say “my car requires 3 ½ litres of oil” rather than “my car requires 7/2 litres of oil”.
- Multiplying and dividing fractions is much easier when they are in improper form.
- Mixed numbers are usually converted to improper fractions before they are used in calculations.

Converting mixed numbers to improper fractions

- To convert from a mixed number to an improper fraction, multiply the whole number by the denominator then add the numerator. The total becomes the new numerator which is then placed over the original denominator. For example:
  - $Convert\ 3\frac{1}{2}\ into\ an\ improper\ fraction.$
  - $working$: $3(whole\ number) \times 2(denominator) + 1(numerator) = 7$ so: $3 \times 2 + 1 = 7$
  - $Therefore\ the\ improper\ fraction\ is\ \frac{7}{2}$

Your turn …

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CONVERTING MIXED NUMBERS TO IMPROPER FRACTIONS

Example problems:

1. $2\frac{2}{3} = \frac{8}{3}$ *Note*: $(2 \times 3 + 2 = 8)$

2. $2\frac{3}{7} = \frac{17}{7}$ *Note*: $(2 \times 7 + 3 = 17)$

Practise problems:

1. $4\frac{1}{2} = \frac{\square}{\square}$

2. $5\frac{1}{3} = \frac{\square}{\square}$

3. $7\frac{3}{5} = \frac{\square}{\square}$

4. $2\frac{1}{8} = \frac{\square}{\square}$

Converting mixed numbers to improper fractions

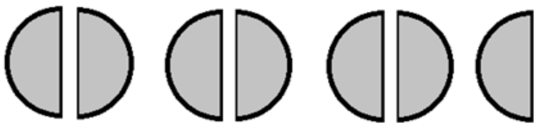
Answers:

1. $4\frac{1}{2} = \frac{9}{2}$
2. $5\frac{1}{3} = \frac{16}{3}$
3. $7\frac{3}{5} = \frac{38}{5}$
4. $2\frac{1}{8} = \frac{17}{8}$

Converting improper fractions to mixed numbers …

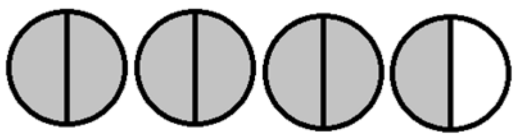
- While improper fractions are good for calculations, they are rarely used in everyday situations.
  - For example, people do not wear a size $\frac{23}{2}$ shoe.
- To convert to an improper fraction we need to work out how many whole numbers we have.
- Let’s use the example from the previous section, in reverse.

Converting improper fractions to mixed numbers …



$= seven\ halves = \frac{7}{2}$

But we can see that 6 of the halves combine to form 3 wholes; with a half left over:



$= 3\frac{1}{2}$

To calculate how many whole numbers we have, we divide the numerator by the denominator. Whatever the remainder is becomes the new numerator!
Using a worked example of the diagram on the previous slide:

$$Convert\ \frac{7}{2} into\ a\ mixed\ number.$$

$Working: 7 \div 2 = 3.5\ \therefore the\ whole\ number\ is\ 3\ with\ some\ remaining.$

$$If\ I\ have\ 3\ whole\ numbers\ that\ is\ 6\ halves\ (3 \times 2).$$

$$I\ must\ now\ have\ 1\ half\ remaining\ (7-6).$$

$$Therefore\ I\ have\ 3\frac{1}{2}$$

Converting improper fractions to mixed numbers …

That was an easy one.

Another example:

$$\text{Convert } \frac{17}{5} \text{ into a mixed fraction.}$$

$$\text{working: } 17 \div 5 = 3.4$$

$$\therefore \quad \text{the whole number is 3 with some remaining.}$$

If I have 3 whole numbers that is 15 fifths. (3

Your turn …

CONVERTING IMPROPER FRACTIONS TO MIXED NUMBERS

Example problems:

1. $\frac{27}{6} = 4\frac{3}{6} = 4\frac{1}{2}$

   $Note: (27 \div 6 = 4.5)\ (4 \times 6 = 24)\ (27 - 24 = 3)\ and\ don't\ forget\ equivalent\ fractions.$
2. $\frac{8}{3} = 2\frac{2}{3}\ Note: (8 \div 3 = 2.67)\ (2 \times 3 = 6)\ (8 - 6 = 2)$

Practise problems:

1. $\frac{7}{5} =$
2. $\frac{12}{9} =$
3. $\frac{53}{9} =$
4. $\frac{27}{7} =$

Converting improper fractions to mixed numbers …

Answers:

1. $\frac{7}{5} = 1\frac{2}{5}$
2. $\frac{12}{9} = 1\frac{3}{9} = 1\frac{1}{3}$
3. $\frac{53}{9} = 5\frac{8}{9}$
4. $\frac{27}{7} = 3\frac{6}{7}$

Converting fractions to decimals

Converting a fraction to decimal form is a simple process - use the divide key on your calculator.

Note: If you have a mixed number, convert it to an improper fraction before dividing it on your calculator.

<u>Example problems:</u>

- $\frac{2}{3} = 2 \div 3 = 0.66666666666etc \approx 0.\dot{6}$
- $\frac{3}{8} = 3 \div 8 = 0.375$ (recap from last week: this decimal terminates)
- $\frac{17}{3} = 17 \div 3 = 5.6666666etc \approx$ 5.67 or 5.$\dot{6}$
- $3\frac{5}{9} = (27 + 5) \div 9 = 3.555555556\,etc \approx$ 3.56 or 3.$\dot{5}$

"Converting fractions to decimals" (and vice versa)
https://www.khanacademy.org/math/pre-algebra/decimals-pre-alg/decimal-to-fraction-pre-alg/v/converting-fractions-to-decimals

Your turn …

CONVERTING FRACTIONS TO DECIMALS

Practise Problems:

1. $\frac{23}{2} =$
2. $\frac{5}{72} =$
3. $56\frac{2}{3} =$
4. $\frac{29}{5} =$

CONVERTING DECIMALS TO FRACTIONS

Practise Problems: (No Calculator first, then check!)

1. 0.65 =
2. 2.666 =
3. 0.54 =
4. 3.14 =

Converting fractions to decimals

Answers:

1. $\frac{23}{2} = 16.5$
2. $\frac{5}{72} = 0.069$
3. $56\frac{2}{3} = \frac{170}{3} = 56.666$
4. $\frac{29}{5} = 5.8$

Converting decimals into fractions …

- Decimals are an almost universal method of displaying data, particularly given that it is easier to enter decimals, as opposed to fractions, into computers.
- However, fractions can be more accurate. For example:
$\frac{1}{3}\ is\ not\ 0.33\ it\ is\ 0.3333333333333333etc!$
- The method used to convert decimals into fractions is based on the notion of place value.
- The place value of the last digit in the decimal determines the denominator.

Converting decimals into fractions ...

Example problems:

- 0.5 has 5 in the tenths column.
  - Therefore 0.5 is $\frac{5}{10} = \frac{1}{2}$ $(don't\ forget\ equivalent\ fractions!)$
- 0.375 has the 5 in the thousandth column.
  - Therefore 0.375 is $\frac{375}{1000} = \frac{3}{8}$
- 1.25 has 5 in the hundredths column ignore the whole number and you have $1\frac{25}{100} = 1\frac{1}{4}$

Converting decimals into fractions ...

Decimal	Fraction
0.125	$\frac{1}{8}$
0.25	$\frac{1}{4}$
0.33333	$\frac{1}{3}$
0.375	$\frac{3}{8}$
0.5	$\frac{1}{2}$
0.66667	$\frac{2}{3}$
.75	$\frac{3}{4}$

Converting decimals into fractions …

- Complete the 'Converting decimals to fractions' practise problems on the handout

Converting decimals into fractions …

Answers:

1. $0.65 = \frac{65}{100} = \frac{13}{20}$
2. $2.666 = 2\frac{666}{1000} = 2\frac{2}{3}$
3. $0.54 = \frac{54}{100} = \frac{27}{50}$
4. $3.14 = 3\frac{14}{100} = 3\frac{7}{50}$

Fraction addition and subtraction

- Adding and subtracting fractions draws on the concept of **equivalent** fractions.
- The golden rule is that you can only add and subtract fractions if they have the **same denominator**

$$\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

- However, if our two fractions do ***not*** have the same denominator, we must use equivalent fractions to find a "**common denominator**" before we can add them.

"Adding and subtraction fractions"
https://www.khanacademy.org/math/arithmetic/fractions/fractions-unlike-denom/v/adding-and-subtracting-fractions

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Fraction addition and subtraction

- We cannot simply add $\frac{1}{4} + \frac{1}{2}$; this will not give us 2 sixths
- These fractions have different denominators (4 and 2). Before these fractions can be added together they must both have the same denominator.
- This can be achieved by making 4 the lowest common denominator, since it is possible to change $\frac{1}{2}$ $into$ $\frac{2}{4}$ by multiplying both the numerator and denominator by two: $\frac{1}{2} \times \frac{2}{2} = \frac{2}{4}$
- So, now we can do the addition because the denominators are the same, and the sum is now: $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ (much bigger than $\frac{2}{6}$)

Fraction addition and subtraction

- Let's look at $\frac{1}{3} + \frac{1}{2} =$
- We cannot simply add these fractions together because the denominators are different.
- So, before we can do the addition, we need to find the **lowest common denominator.**
- The easiest way to do this is to multiply the denominators: $\frac{1}{3}\, and\ \frac{1}{2}$ (2 x 3 = 6).
- Therefore, both fractions can have a denominator of 6.

Fraction addition and subtraction

The next step is convert both fractions into their equivalent form, with a 6 as the denominator:

- what do we do to $\frac{1}{3}$ $to\ convert\ to$ $\frac{?}{6}$

$$\Rightarrow \frac{1 \ \times \mathbf{2}}{3 \ \times \mathbf{2}} \ which\ is\ \frac{2}{6}$$

- and:

What do we do to $\frac{1}{2}$ $to\ convert\ to$ $\frac{?}{6}$

$$\Rightarrow \frac{1 \times \mathbf{3}}{2 \times \mathbf{3}} \ which\ is\ \frac{3}{6}$$

Fraction addition and subtraction …

- So, the sum becomes:

$$\frac{1}{3}+\frac{1}{2}= \qquad \frac{2}{6}+\frac{3}{6} \quad =\frac{5}{6}$$

Fraction addition and subtraction

- With practise, a pattern forms, as is illustrated in the next example:
- $\frac{1}{3}+\frac{2}{5}= \frac{(1\times5)+(2\times3)}{(3\times5)}= \frac{5+6}{15}=\frac{11}{15}$
- In the example above, the **lowest common denominator** is found by multiplying 3 and 5, then the numerators are multiplied by 5 and 3 respectively.

Fraction addition and subtraction

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FRACTION ADDITION AND SUBTRACTION

Example addition problems:

1. $\frac{3}{4}+\frac{2}{7}=\frac{(3\times7)+(2\times4)}{(4\times7)}=\frac{21+8}{28}=\frac{29}{28}=1\frac{1}{28}$
2. $2\frac{2}{3}+1\frac{3}{4}=\frac{8}{3}+\frac{7}{4}=\frac{(8\times4)+(7\times3)}{(3\times4)}=\frac{32+21}{12}=\frac{53}{12}=4\frac{5}{12}$ Note: Convert mixed to improper.

Subtraction is the same procedure with a negative symbol.

$$\frac{2}{3}-\frac{1}{4}=\frac{(2\times4)-(1\times3)}{(3\times4)}=\frac{8-3}{12}=\frac{5}{12}$$

Example subtraction problems:

1. $\frac{9}{12}-\frac{1}{3}=\frac{(9\times3)-(1\times12)}{(12\times3)}=\frac{27-12}{36}=\frac{15}{36}=\frac{5}{12}$ Note: equivalent fraction
2. $\frac{1}{3}-\frac{1}{2}=\frac{(1\times2)-(1\times3)}{(3\times2)}=\frac{2-3}{6}=\frac{-1}{6}=-\frac{1}{6}$ Note: you have taken more then you originally had.

Practise Problems:

1. $\frac{1}{3}+\frac{2}{5}=$
2. $2\frac{1}{6}+3\frac{7}{8}=$

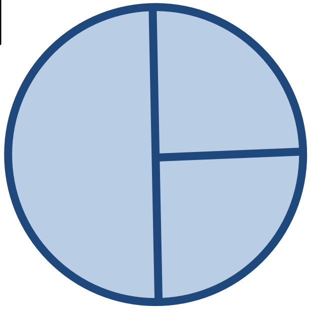
Fraction addition and subtraction

Answers:

1. $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$
2. $2\frac{1}{6} + 3\frac{7}{8} = 6\,\frac{1}{24}$

Fraction multiplication and division

- Compared to addition and subtraction, multiplication and division of fractions is easy to do, but rather odd to understand.
- For example, imagine I have $\frac{1}{2}$ of a pizza and I want to share it between 2 people.



- Each person gets a quarter of the pizza.
- Mathematically, this example would be written as:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}.$$

Fraction multiplication and division

Tip:

Remember that fractions and division are related so multiplying by a half is the same as dividing by two.

“Multiplying negative and positive fractions”
https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-fractions-decimals/cc-7th-mult-div-frac/v/multiplying-negative-and-positive-fractions

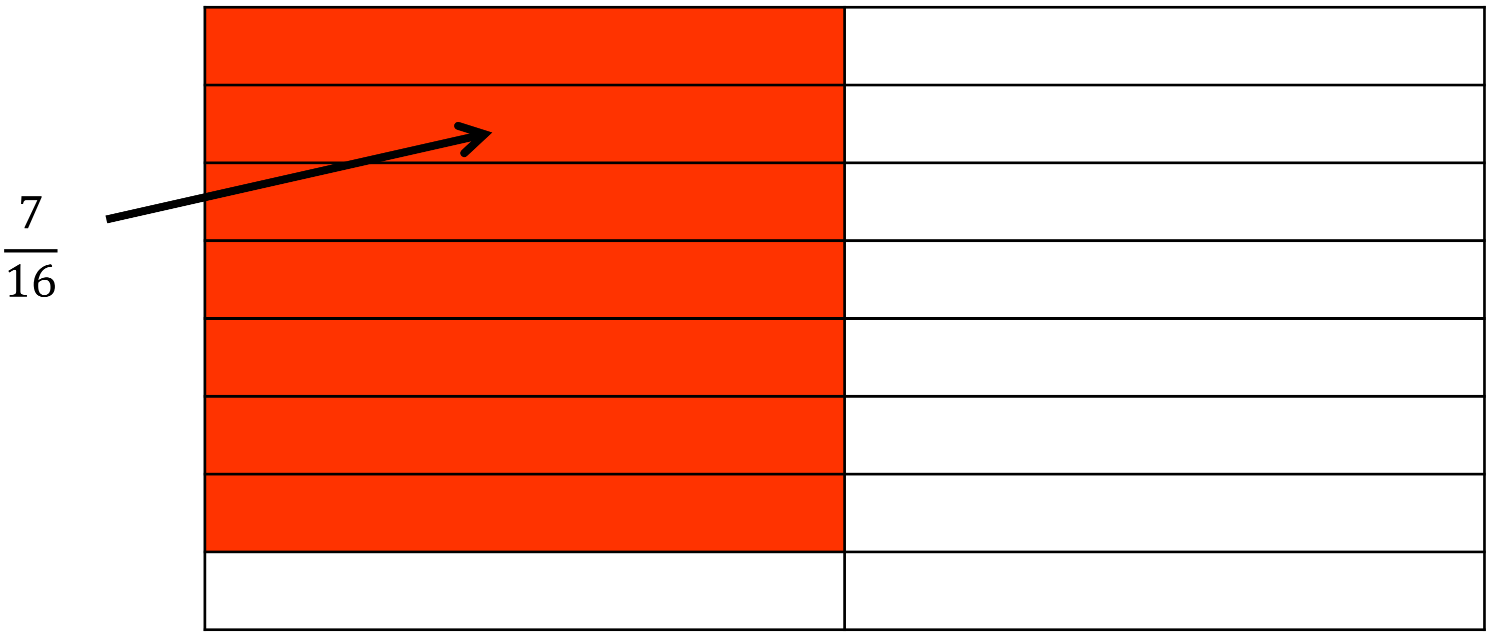
“Dividing fractions example”
https://www.khanacademy.org/math/arithmetic/fractions/div-fractions-fractions/v/another-dividing-fractions-example

Fraction multiplication and division

- But what if the question is more challenging: $\frac{2}{3} \times \frac{7}{16}$
- This problem is not as easy as splitting a pizza!
- The rule to use is: "multiply the numerators then multiply the denominators"
- Therefore $\frac{2}{3} \times \frac{7}{16} = \frac{(2 \times 7)}{(3 \times 16)} = \frac{14}{48} = \frac{7}{24}$

Fraction multiplication and division

We have a whole rectangle and the orange section represents $\frac{7}{16}$ of the whole,

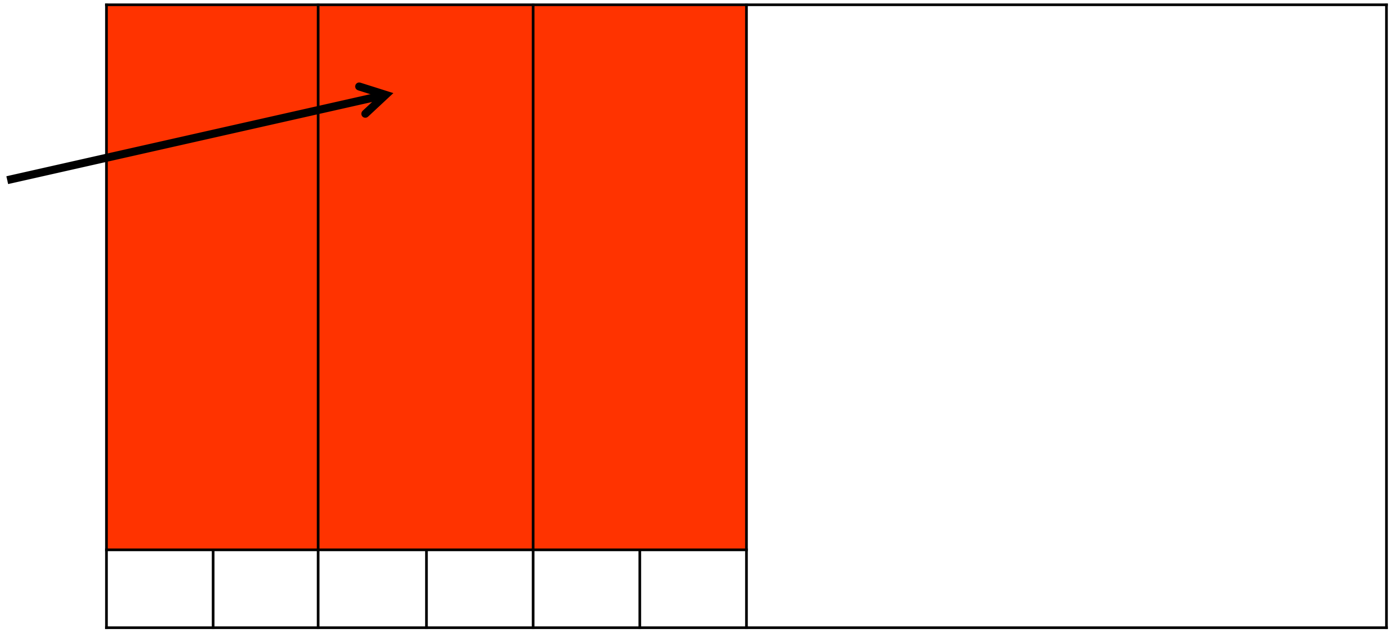


$$\frac{2}{3} \times (of)\, \frac{7}{16} = \frac{7}{24}$$

Fraction multiplication and division

We have a whole rectangle and the orange section represents $\frac{7}{16}$ of the whole, then we divide that section into thirds,

$$\frac{7}{16}$$

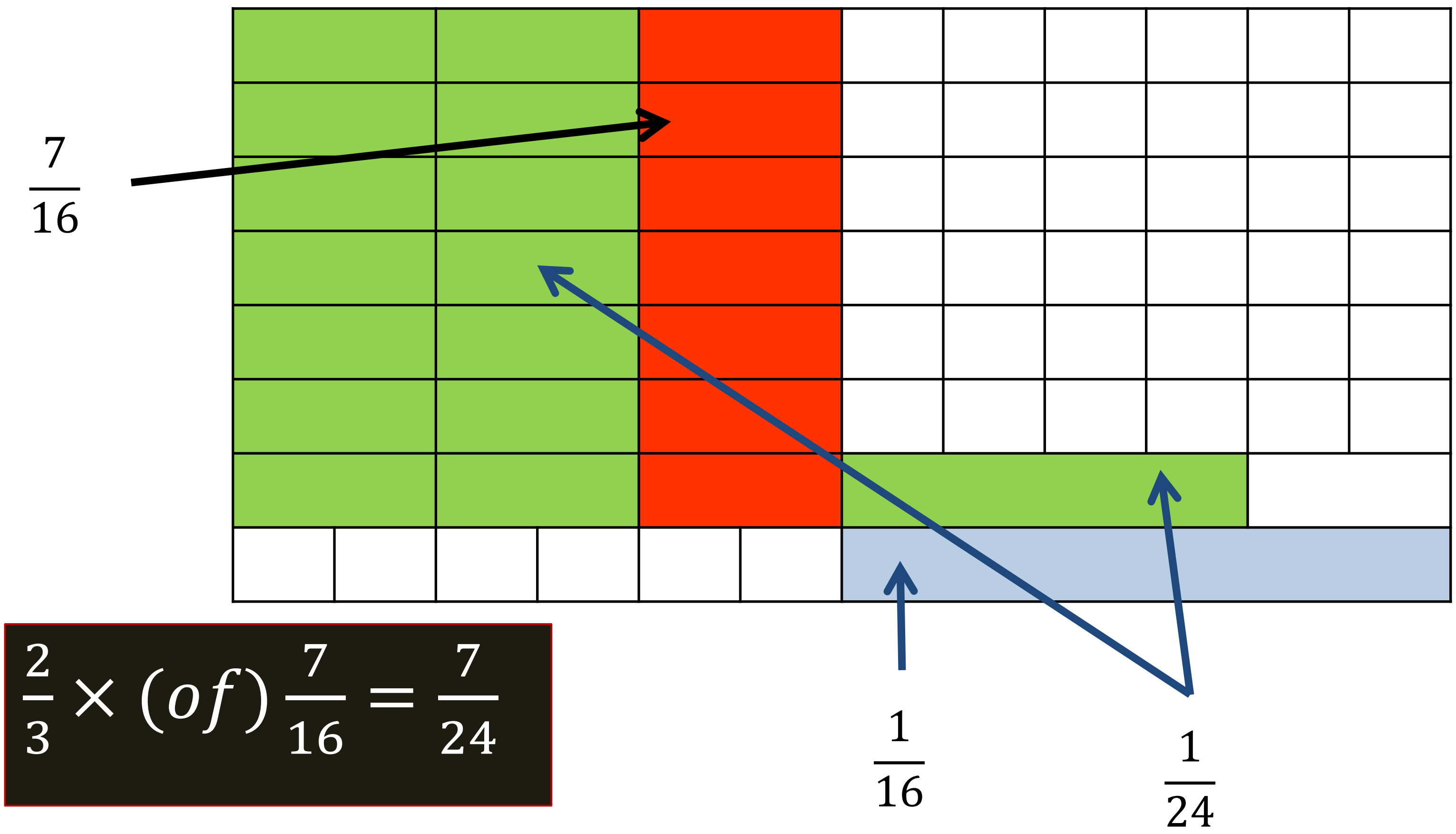


$$\frac{2}{3} \times (of)\, \frac{7}{16} = \frac{7}{24}$$

Fraction multiplication and division

We have a whole rectangle and the orange section represents $\frac{7}{16}$ of the whole, then we divide that section into thirds,

then we can see we have 7 sections that are each $\frac{1}{24}$; therefore $\frac{7}{24}$



$$\frac{2}{3} \times (of)\, \frac{7}{16} = \frac{7}{24}$$

Fraction multiplication and division

We have a whole rectangle and the orange section represents $\frac{7}{16}$ of the whole, then we divide that section into thirds,

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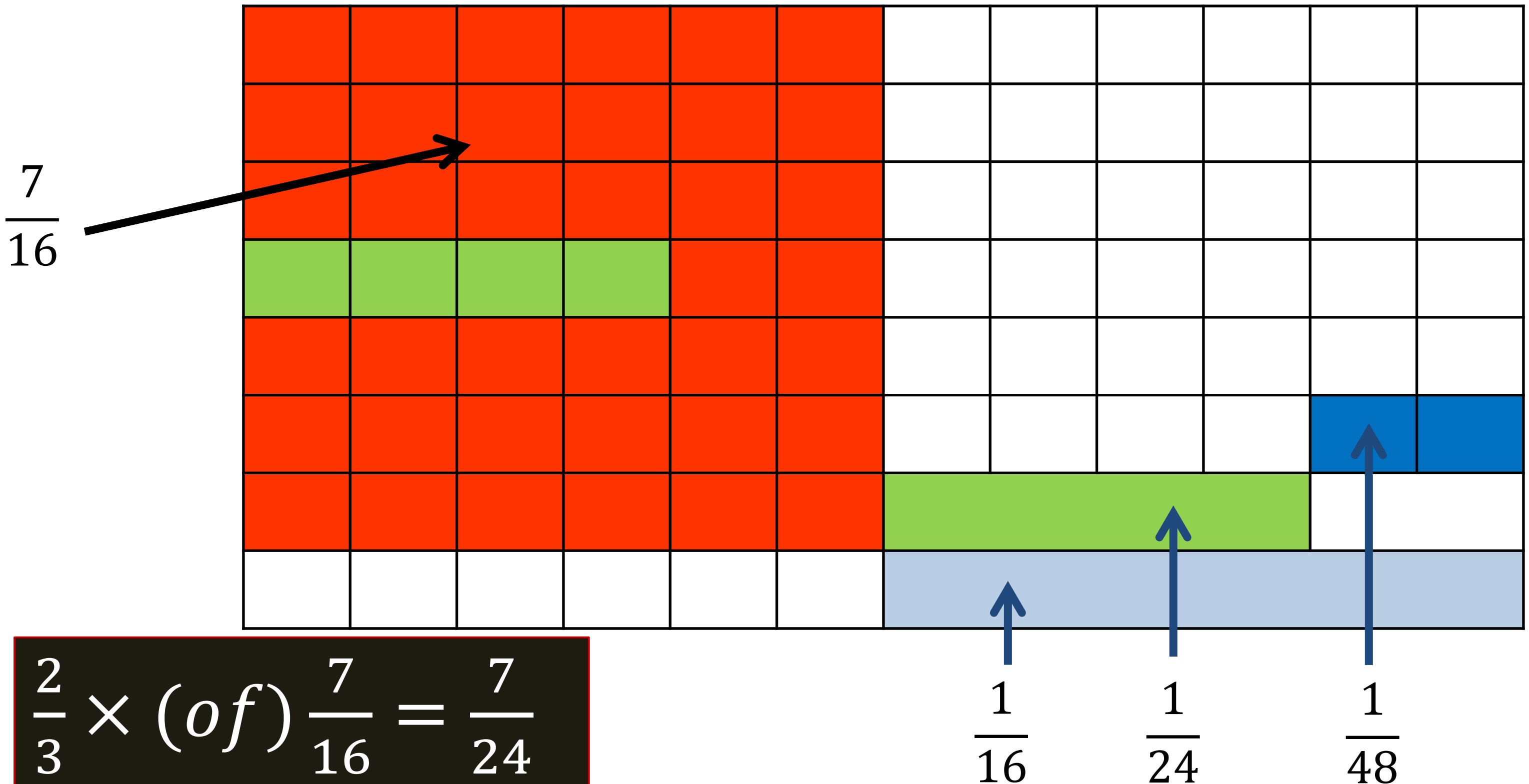


$$\frac{2}{3} \times (of)\, \frac{7}{16} = \frac{7}{24}$$

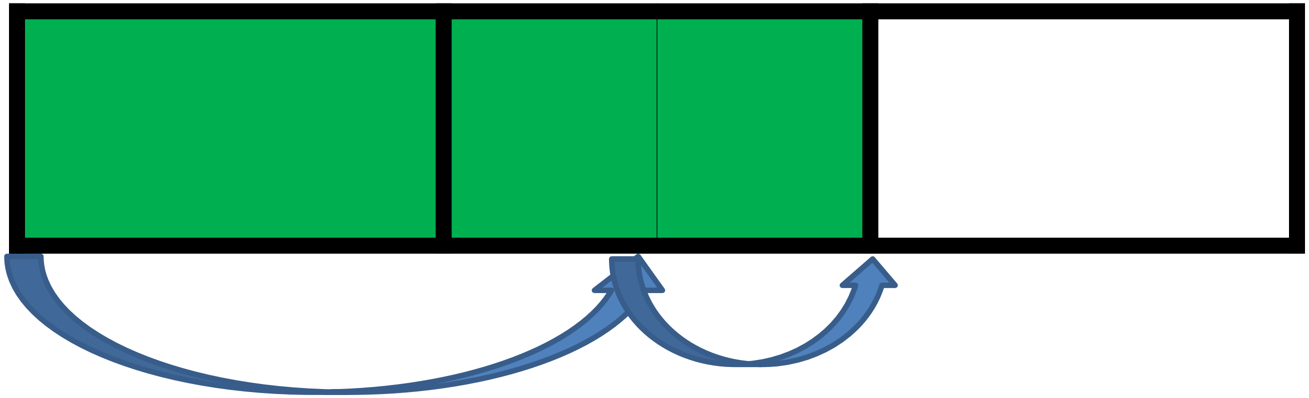
Fraction multiplication and division

At first, division of fractions seems odd, but it is a simple concept:

$$\div \frac{1}{2} \; is \; the \; same \; as \; \times \frac{2}{1}$$

- If the sign is swapped to its opposite, the fraction is flipped upside down (the fraction that is "flipped upside down" is referred to as the **reciprocal)**.
- Therefore $\frac{2}{3} \div \frac{1}{2}$ $is \; the \; same \; as$ $\frac{2}{3} \times \frac{2}{1} = \frac{(2\times2)}{(3\times1)} = \frac{4}{3} = 1\frac{1}{3}$ ***Note***: dividing by half doubled the answer.

Fraction multiplication and division



We can ask how many halves can we take out of two thirds. There is one half and then one third of a half.

$$\frac{2}{3} \div \frac{1}{2} = 1\frac{1}{3}$$

Your turn….

FRACTION MULTIPLICATION AND DIVISION

Example Multiplication Problems:

1. $\frac{4}{9} \times \frac{3}{4} = \frac{(4\times3)}{(9\times4)} = \frac{12}{36} = \frac{1}{3}$
2. $2\frac{4}{9} \times 3\frac{3}{5} = \frac{22}{9} \times \frac{18}{5} = \frac{(22\times2)}{(1\times5)} = \frac{44}{5} = 8\frac{4}{5}$

Example Division Problems:

1. $\frac{2}{3} \div \frac{3}{5} = \frac{2}{3} \times \frac{5}{3} = \frac{(2\times5)}{(3\times3)} = \frac{10}{9} = 1\frac{1}{9}$
2. $3\frac{3}{4} \div 2\frac{2}{3} = \frac{15}{4} \div \frac{8}{3} = \frac{15}{4} \times \frac{3}{8} = \frac{(15\times3)}{(4\times8)} = \frac{45}{32} = 1\frac{13}{32}$

Practise Problems:

1. $\frac{2}{3} \times \frac{7}{13} =$
2. $1\frac{1}{6} \times \frac{2}{9} =$
3. $\frac{3}{7} \div \frac{2}{5} =$
4. $2\frac{2}{5} \div 3\frac{8}{9} =$

Fraction multiplication and division

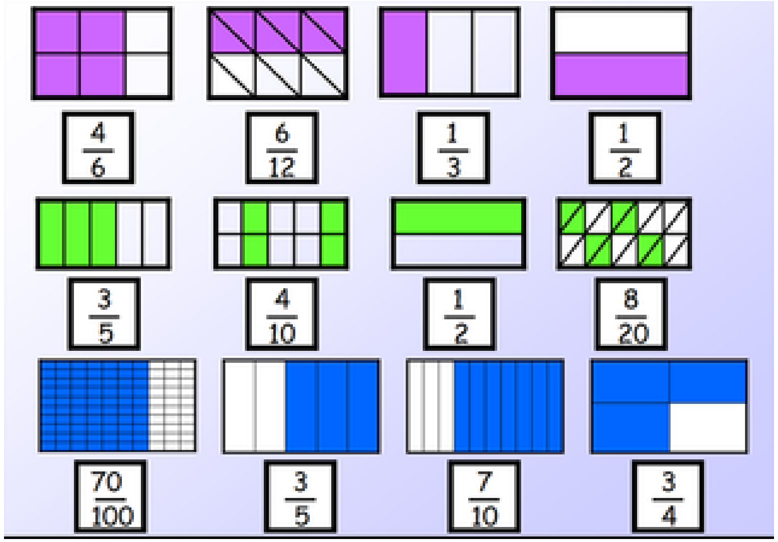
Answers:

1. $\frac{2}{3} \times \frac{7}{13} = \frac{14}{39}$
2. $1\frac{1}{6} \times \frac{2}{9} = \frac{7}{27}$
3. $\frac{3}{7} \div \frac{2}{5} = 1\frac{1}{14}$
4. $2\frac{2}{5} \div 3\frac{8}{9} = \frac{108}{175}$

Working with fractions

Reflect on the learning intentions ….

- Become familiar with fractions
- Equivalent fractions
- Converting mixed numbers to improper fractions
- Converting improper fractions to mixed numbers
- Converting decimals into fractions
- Converting fractions into decimals
- Fraction addition and subtraction
- Fraction multiplication and division



http://www.3to11maths.com/equivalent-fractions.html