Everything You Always Wanted to Know About SFC Modelling* (*But Were Afraid to Ask)

Introduction

SFC models are purely macroeconomic, since so-called “microfoundations” are not explicitly used to derive macroeconomic behavioural equations. It is assumed that behavioural equations for groups of agents are defined using aggregate values such as the GDP, disposable income or the stock of wealth.

The roots of SFC modelling can be found in the work of Tinbergen (1940). A discrete-time model presented in the paper can be described as a system of algebraic-difference equations. Kalecki (1954) used a model of a closed economy in defined terms of a balance sheet to derive his profit equation. A dynamic model of the economic cycle in a form of a difference equation was also presented in the same book. A dynamic model of an economy was simulated using an analogue hydraulic computer (MONIAC) by AWH Phillips (Leeson, 2011). Phillips also wrote two papers (included in his “Collected Works”, edited by Leeson, 2011) on using Optimal Control methods to stabilise an economy. Godley and Lavoie (2007) present a “stock-flow consistent” Post-Keynesian modelling framework, by putting emphasis on the accumulation and balance-sheet consistency aspects of dynamic macroeconomic models.

It is not true that mainstream (neoclassical and New Keynesian) models need to be stock-flow inconsistent, some of them are stock-flow consistent. The name “SFC” is misleading as the main feature of the methodology is the rejection of neoclassical microfoundations and certain structural elements such as the loanable funds market or a production function of a certain shape (e.g. Cobb-Douglas). While neoclassical models assume the presence of a rational representative agent (or a few of them), maximising the utility function in a predictable macroeconomic environment (with the accumulation path determined by the effects of this utility maximisation process), Post-Keynesian models are based on the observation that developed real world economies are demand-driven in the short run. There is focus on capital accumulation in the medium-run, while in the long-run the growth of productivity and the workforce determine the evolution of the system. Productivity growth can also be endogenised as it depends on the level of spending on education, science and specific R&D projects (often related to national defence).

Modern SFC models are built on similar foundations as “old-Keynesian” dynamic models which fell out of fashion after the Lucas critique (1976). The main issue with the neoclassical approach is that microfoundations do not describe human behaviour correctly, there is a significant fraction of agents maximising their stock of wealth subject to satisfy current consumption needs, not just optimising the intertemporal path of consumption in order to maximise the total utility function. These agents as a social class succeed in harvesting fruits of the labour of workers, who have “hand-to-mouth” consumption spending behaviour. Average saving rates of corresponding income groups cannot be explained by any microfounded models as the rich accumulate much more that they would need in order to save for a rainy day, even assuming the highest plausible level of risk aversion (affecting so-called liquidity preference). These rich agents are simply “greedy”, as they save for the sake of getting even richer and the volume of their savings (the flow) is detached from the needs of the economy for investment funds. In this context the average liquidity preference of the richest is not only determined by the need to have liquid “buffers” but is also an artefact of how much of the current disposable income can be invested in productive capital – the rest simply remains unspent in the form of money and its close equivalents. The shape of the aggregate consumption function (which can be determined either for separate social classes or for the whole household sector) is determined by these factors. In turn, it is the aggregate consumption function and exogenous injections and leakages which determine the level of the GDP, capacity utilisation and employment in the short run. Consumption is assumed to depend (in the simplest case) on the current disposable income and the expected stock of wealth. The economy undergoes quantity adjustments while prices are sticky in the short run and for the most of final goods and services, determined by normal costs and the prevailing rate of profit. The main factor determining the volume of planned investment is the expected level of capacity utilisation. The long run is nothing else but a sequence of short runs as stated by Kalecki (1971, p.165).

More sophisticated SFC models include the main institutional components of an economy. The firms sector usually produces a single good with a non-neoclassical production function. Several production sectors can be introduced. It is often assumed that the volume of production is a function of expended labour while the stock of fixed capital that is required for the production process to occur is always sufficient. This assumption corresponds to a Leontief-type production function operating in the region constrained by the labour input. It is possible to introduce more complex production functions. Unemployment is considered to be caused by inadequate aggregate demand, not firms reducing their demand for labour if unit wages increase due to an excessive bargaining power of trade unions, while the supply of labour increases with unit wages.

SFC models use a form of markup pricing to determine unit prices of abstract goods. Modelling of inflationary processes is possible due to the presence of a unit price or a CPI (Consumers Price Index) if multiple final commodities exist. As a consequence, two sets of variables, real and nominal, need to be defined in the model. Inflation is not assumed to be a direct effect of excessive aggregate demand or the rate of unemployment lower than NAIRU but rather a product of conflicting claims on the share of output between the labour and owners of firms (capitalists).

The government sector plays a crucial role in redistributing income, stabilising aggregate demand and providing financial assets to households (other roles of the government, such as providing public goods and services, may be hidden unless explicitly implemented in the model).

The financial sector usually consists of the central bank (which is functionally a part of the government sector) and commercial banks. Money in SFC models is endogenous, so the fallacy of the Loanable Funds Theory is avoided. Banks create money by extending loans to firms and households. Net lending may contribute to aggregate demand in a way similar to government deficit spending. Multiple asset classes can be introduced to SFC models, these include money, government securities, equities and real estate. An asset pricing subsystem can be implemented by using the Tobin portfolio model.

SFC models can accommodate the foreign sector or may span several countries. Foreign denominated assets and liabilities may be present on the balance sheets of the economic sectors. It is assumed that both the purchasing power parity and the uncovered interest parity conditions do not hold.

Internal consistency of SFC models

A model is defined as a system of linked balance sheets and transaction-flow matrices, derived from the accounting principle of quadruple entry. Balance sheets and transaction-flow matrices correspond to ex-post identities and define structural components of the model. Magnitudes of flows are determined by behavioural equations. Real and nominal flows may depend on the values of expectational parameters, which correspond to the state of current knowledge about the economy by the agents.

Accounting consistency of the model is preserved on flow, stock and stock-flow levels (stock level consistency is implied by flow and stock-flow consistency if initial values of stocks are consistent). Behavioural components of the model are usually "ex-ante" approximations of what may happen in the economy, based on the most likely scenario. Behavioural equations are by definition based on simplified assumptions. The structure of the model is also simplified. For example, the economy may only produce a single good and several categories of financial assets (bonds, bills) may be lumped together. These simplifications (behavioural and structural) are the trade-off required to build a macro model which can still be managed and yet produces useful results.

Steps required to build an SFC model

Usually building an SFC model (or modifying an existing one) requires choosing which sectors are to be included, defining stocks and flows, then creating a balance sheet and a flows matrix. Once the structural components (ex-post identities) are defined, behavioural equations can be added.

Usually (as presented by Godley and Lavoie, 2007), only net assets are shown in a balance sheet but showing both assets and liabilities of a sector may be beneficial as equity is then explicitly shown as the difference between assets and liabilities.

Table 1: Balance sheet of model BMW (Godley and Lavoie, 2007, p. 219)

	Households		**Production firms**		**Banks**	
	Assets	**Liabilities + Equity**	**Assets**	**Liabilities + Equity**	**Assets**	**Liabilities + Equity**
Money deposits	M					M
Loans				L	L	
Fixed capital			K			
Equity		V_h				

Godley and Lavoie (2007) present separate transactions-flow and behavioural transaction matrices. No new information about the model structure is contained in a behavioural transaction matrix so it may be more convenient to present behavioural equations separately, while defining volumes of the flows depicted in the flow matrix. Sums of columns and rows of the matrix must be equal to zero as inflows (recorded with the positive sign) match outflows (recorded with the negative sign). Some flows correspond to changing stocks. This is the stock-flow consistency element of the model.

Table 2: Transaction flow matrix of model BMW (Godley and Lavoie, 2007, p. 220)

	Households	Production firms		Banks
		Current	Capital	
Consumption	-C	C		
Investment		I	-I	
Wages	WB	-WB		
Capital depreciation		-DA	DA	
Interests on loans		-INT		INT
Interests on deposits	-INT			-INT
Change in loans			ΔL	-ΔL
Change in deposits	-ΔM			ΔM

Structural equations define values of several variables, for example in this model the rate of interests on loans and deposits must be equal as it is assumed that banks have zero net worth (the sum of the last column in the transaction flow matrix is zero).

Godley and Lavoie (2007) also redefine flows in terms of demand and supply but this does not contain any additional information about the model. Then several auxiliary variables, such as the gross domestic product ("Y") or disposable income ("YD") are introduced. Finally, the model is closed by providing several behavioural equations, defining the behaviour of agents (aggregated into sectors of the economy). In model BMW (Godley and Lavoie, 2007, Chapter 7), these behavioural equations define:

- capital depreciation (amortisation funds) DA,
- loans extended to firms ΔL,
- consumption of the households C,
- gross investment I (determined as a function of the GDP, "Y", previous period stock of capital, "K" and capital depreciation "DA").

Household consumption is a function of the current period disposable income and previous period wealth (household deposits). Assuming that gross investment coefficients (which depend on the technique of production) are constant, it is the consumption function which determines the value of the GDP flow (Godley and Lavoie, 2007, p.229).

The general mathematical form of discrete-time SFC models

Discrete-time models are usually defined by providing equations determining values of all variables

$$\boldsymbol{X_t} = \boldsymbol{F}(t, \boldsymbol{X_{t\text{-}1}}, \boldsymbol{X_t}) \tag{1}$$

where $\mathbf{X}_t$ is an n × 1 vector of variables X at a time index "t" and $\mathbf{X}_0$, the vector of initial values, is provided. In some cases the system is linear however the majority of more complex models are non-linear.

Equation (1) is a special case of a more generic class of systems of algebraic-difference equations defined as:

$$\boldsymbol{F}(t, \boldsymbol{X_{t\text{-}1}}, \boldsymbol{X_t}) = \mathbf{0} \tag{2}$$

Both equations (1) and (2) define first order systems of equations.

More information (with references) about a similar class of problems, defined in continuous-time (algebraic-differential equations) can be found in Campbell, Linh & Petzold, (2008).

The problem of simulating a model defined by equation (1) can be presented as an iterative process of solving a system of nonlinear algebraic equations for every time step "t=1,2,…" knowing values from the previous step, "t-1". The iterative process is initiated by solving the system at "t=1" using known initial values at "t=0". In a special case, the system of equations may be linear. Solving the system of nonlinear algebraic equations at the time "t" is usually done using an iterative numerical method itself. If the model is defined incorrectly, the evolution of the algebraic-difference system of equations may lead to an explosive or decaying trajectory. Despite this, the model can always be simulated as long as the solver converges for every time step. Unfortunately, solving the system of nonlinear equations at every time step may be difficult. Finding the initial values at a time t=0 is usually the most challenging task.

Simulating a simple model using a spreadsheet

For some users, setting up a spreadsheet (Microsoft Excel, LibreOffice Calc) appears to be the easiest way of simulating a model. This may work in some cases but it is likely to fail with nonlinear models, unless macros are used. Bernardo (2012) implemented in Microsoft Excel a collection of models defined by Godley and Lavoie (2007).

Setting up a spreadsheet involves defining the values of exogenous parameters (as shown in Figure 1) and providing equations recursively defining values of endogenous variables (Figure 2). These equations cannot be defined using symbols of variables but rather cell indexes. This makes analysing, modifying and debugging a model difficult.

	A	B	C	D
1				
2		**Parameters**	**Symbols**	**Values to change**
3		Tax rate	θ	20%
4		Propensity to consume (income)	α_1	0.6
5		Propensity to consume (wealth)	α_2	0.4
6		Public expenditures	G	20

Figure 1: Exogenous parameters of ModelSIM.xls (Bernardo, 2012).

F	G	H	I	J
Variables/Time	**Symbols**	**1957**	**1958**	**1959**
Public expenditures	G	-	=$D$6	=$D$6
National Income	Y=C+G	-	=I7+I3	=J7+J3
Taxes	"T=θ.Y"	-	=I4*$D$3	=J4*$D$3
Disposable Income	YD=Y-T	-	=I4-I5	=J4-J5
Consumption	"C=(α1.YD)+(α2.H-1)"	-	=($D$4*I6)+(H10*$D$5)	=($D$4*J6)+(I10*$D$5)
Change in cash (Gov.)	ΔH	-	=I3-I5	=J3-J5
Change in cash (hous.)	ΔH	-	=I6-I7	=J6-J7
Wealth	H=ΔH	-	=I8+H10	=J8+I10

Figure 2: Equations defining ModelSIM.xls (Bernardo, 2012).

The model is then simulated by the spreadsheet software. Since some variables defined in a particular column depend on other variables defined in the same column (variables defined for time step "t" depend not only on values of variables at "t-1" but also current "t"), iterative evaluation (affecting the whole column of the table), needs to be enabled in the spreadsheet options. In Microsoft Excel this is enabled in "File > Options > Formulas > Preferences > Calculation" while in LibreOffice this option can be found in "Tools > Options > LibreOffice Calc > Calculate". If iterative evaluation is not enabled, "Error 522" is shown as the spreadsheet program fails to solve the system of algebraic equations. However even if iterative evaluation is enabled, there is still no guarantee that the process of finding the values in a particular column will converge.

The iterative process is effectively an implementation of the fixed-point iteration method. In a spreadsheet, values of multiple variables contained in the current column may initially be initialised to zero, if these variables cannot be calculated from the previous column values. In the model presented in Figure 2, current consumption depends on the stock of wealth in the previous period and current disposable income, current disposable income depends on the current GDP and current taxes, taxes depend on the GDP, GDP depends on the consumption and (exogenous) government expenditure. There are clearly circular dependencies in our example and these variables, which are unknown (for example current disposable income used to calculate current consumption), are initially set to zero. This is the starting point of the iterative process which may be outside of the region of convergence, even if a fixed point exists and there is a convergence region around the solution. We cannot provide a different starting point (for example values of corresponding variables from the previous time step), unless a macro or a so-called add-in is used. It would be even possible to run one of the well-known numerical methods on spreadsheet data. But if we need to start writing spreadsheet macros or add-ins while still dealing with the opaqueness of the spreadsheets, not allowing us to use symbols in the formulas, it may be just easier to write the whole implementation of the model in a general-purpose or domain-specific programming language (Python, Java, Julia, R, Matlab). For an economist without programming experience, the use of commercial software (Eviews) may be the best option, as it uses a quite sophisticated algorithm for solving systems of nonlinear equations (Brillet, 2011).

Fixed-point iteration method

Multiple textbooks on numerical methods for example these written by Hoffman (2001) and Zarowski (2004) cover the topic of solving nonlinear equations and systems of nonlinear equations. The fixed-point iteration method is applicable to equations in the following form:

$$x=g(x) \tag{3}$$

If an equation is defined as:

$$f(x)=0 \tag{4}$$

then the function needs to be rearranged to the form required in equation (3)

$$g(x)=f(x)+x \tag{5}$$

The fixed-point iteration method requires knowing an initial estimation of the solution, x_0. Then the following iterative algorithm is applied:

$$x_{i+1}=g(x_i) \tag{6}$$

The procedure is repeated until a convergence criterion is met (as described in the textbooks).

The algorithm starting from initial value x_0 will converge to a solution p if the function g(x) is contractive in the range defined by x_0 and p. This condition defined for range [a,b] means that for any x,y ∈ [a,b] there exists a real number α so that:

$$|g(x)-g(y)|\leq \alpha|x-y|, 0<\alpha<1 \tag{7}$$

The condition is often expressed by stating that the absolute value of the derivative g'(x) of g(x) must be less than 1 for every x ∈ [a,b].

The solution p is a fixed point of the relationship (mapping)

$$p=g(p) \tag{8}$$

The fixed-point iteration method is based on Banach's fixed-point theorem (Zarowski, 2004, p. 297). Other numerical methods used for solving nonlinear equations also rely on the same theorem but a different transformation of the function f(x), defined in equation (4) is used. For example (Zarowski, 2004, p. 307), the well-known Newton-Raphson method uses the following function g(x)

$$g(x)=x-\frac{f(x)}{f'(x)} \tag{9}$$

The Newton-Raphson method may still converge in these cases when the original fixed-point iteration method is divergent.

The fixed-point method can also be applied to systems of nonlinear equations (Zarowski, 2004, pp. 313-318). The convergence criterion is similar.

$$\|\boldsymbol{G}(\boldsymbol{x})-\boldsymbol{G}(\boldsymbol{y})\|\leq \alpha\|\boldsymbol{x}-\boldsymbol{y}\|, 0<\alpha<1 \tag{10}$$

where **x, y** $\in R$, *R* is typically defined as a bounded and closed rectangular region.

The convergence of the method can be proven using the Banach's fixed-point theorem. The intuitive meaning of equation (10) is that the norm of the function should change slower than the norm of its argument in the region where the method is convergent. The starting point of the iterative process should belong to the region. Unfortunately, the fixed-point iteration method is not convergent for many non-linear systems of equations describing SFC models, when the starting point of the process is a zero vector. This is the reason why a spreadsheet without macros cannot be used for simulating these models.

This problem can be addressed by using a method which has a larger region of convergence (usually one of the gradient methods with numerical differentiation) and supplying a suitable starting point. Initial values of stocks have to be defined anyway. "Plausible" initial values of other variables need to be assumed or guessed. These values may need to be manually changed until the model converges. Once the initial vector (at t=0) is calculated, all variables evaluated at the current time step (at t=1,2,…) may be initialised to the values from the previous time step t-1 as the modelled economy only slowly evolves or reacts to limited exogenous shocks. These initial values are usually close enough to the fixed point (solution) of the system of equations to provide convergence of the numerical method used to solve the system of algebraic equations.

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